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-+-SOME COMMON FIXED POINT THEOREMS IN FUZZY METRIC SPACES USING IMPLICIT RELATION AND THE PROPERTY (E.A)

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ABSTRACT
In this paper, we establish some common fixed point theorems in fuzzy metric spaces for sequence of self mappings using implicit relation and the property (E.A). Our results extend, generalize and improve several results of metric spaces and menger spaces to fuzzy metric spaces.

KEYWORDS: Fuzzy metric space, t-norm, weakly commuting mappings, (E. A) property, implicit relation .
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1. INTRODUCTION

The idea of fuzzy set was first introduced by Iranian mathematician (L.A. Zadeh,1965). Fuzzy set is specifically designed to mathematically represent uncertainty and vagueness. Fuzzy set is defined by a membership function which assigns each object to a grade of membership between zero and one. (Kramosil and Michalek, 1975) applied the concept of fuzziness to the classical metric space and compare the fuzzy metric space with probabilistic metric space, the generalization of metric space. (George and Veeramani,1994) modified the concept of fuzzy metric space by imposing some stronger conditions using continuous t-norm and defined the hausdorff topology of fuzzy metric spaces. (Gregori and Sapena, 2002) defined the concepts of convergent sequence, cauchy sequence, completeness and compactness in sense of fuzzy metric space. (Grabiec, 1989) introduced the fuzzy version of Banach contraction principle.

(Jungck, 1976) introduced commuting mappings in metric space. Sessa (1986) generalized commuting mappings in metric space as weakly commuting mappings. Pant (1994) introduced R weak commutativity in metric space. Vasuki (2010) defined R weak commutativity in fuzzy metric space. Jungck (1986) enlarged the class of noncommuting mappings by compatible mappings. Also the concept of compatible mappings was further improved by Jungck et al. (2006) with the notion of weakly compatible mappings which merely commute at coincidence points. Mishra et al. (1991) defined compatible mappings in fuzzy metric spaces and proved some common fixed point theorems.

Aamri and El Moutawakil (2002) defined the (E.A) property for self mappings whose class contains the class of noncompatible as well as compatible mappings. Common property (E.A) is introduced by Ali et al. (2010). It is observed that (E.A) property and common property (E.A) require the closedness of the subspaces for the existence of fixed point. Mihet (2010) defined (E. A) property in fuzzy metric spaces.
In this paper, we prove some common fixed point theorems in fuzzy metric spaces using implicit relation and the common property (E. A).

2. Preliminaries:

Definition 2.1 (Schweizer et al. 1986): A binary operation $*: [0,1] \times [0,1] \rightarrow [0,1]$ is called continuous t-norm if it satisfies the following conditions:

1) $*$ is commutative and associative;
2) $*$ is continuous;
3) $a * 1 = a$, for all $a \in [0, 1]$;
4) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, for all $a, b, c, d \in [0, 1]$

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Remark 2.2 (Schweizer et al. 1986): The concept of t-norm can be considered as fuzzy union.

Definition 2.3 (George et al. 1994): The 3-tuple (X, M, ∗) is said to be a fuzzy metric space (FMS) if, X is a non empty set, ∗ is a continuous t-norm, M is a fuzzy set on $X \times X \times (0, \infty)$ satisfying the following conditions:

1) $M(x, y, t) > 0$;
2) $M(x, y, t) = 1$ if and only if $x=y$;
3) $M(x, y, t) = M(y, x, t)$;
4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$;

$M(x, y, \cdot): (0, \infty) \rightarrow [0, 1]$ is continuous, for all x, y, z ∈ X and s, t > 0.

M(x, y, t) is considered as the degree of nearness of x and y with respect to t.

Example 2.4 (George et al. 1994):
Let (X, d) be a metric space $a * b = \min\{a, b\}$ and ∀ x, y ∈ X and t > 0.

$$M_d(x, y, t) = \frac{t}{t + d(x, y)},$$

then (X, M, ∗) is a Fuzzy metric space.

Definition 2.5 (Gregori et al. 2002):

I. A sequence $\{x_n\}$ is said to convergent to x in X, if and only if $\lim_{n\to\infty} M(x_n, x, t) = 1$ for all t>0.
II. A sequence $\{x_n\}$ is said to M cauchy, if and only if for each $\varepsilon \in (0,1)$, $t > 0$, there exist $n_0 \in N$ such that $\lim_{n\to\infty} M(x_m, x_n, t) > 1 - \varepsilon$ for any $m, n \geq n_0$ for all t>0.
III. The fuzzy metric space M(x, y, t) is called M-complete if every M-cauchy sequence is convergent.

Definition 2.5 (Vasuki 1999): Two self mappings f and g of a fuzzy metric space (X, M, ∗) are said to be weakly commuting if,

$$M(fgx, gfx, t) \geq M(fx, gx\ t), \text{ for all } x \in X, t > 0.$$

Definition 2.5 (Vasuki 1999): Two self mappings f and g of a fuzzy metric space (X, M, ∗) are said to be R weakly commuting if,

$$M(fgx, gfx, t) \geq M(fx, gx, \tfrac{t}{R}), \text{ for all } x \in X, t > 0.$$

Definition 2.8 (Mihet 2010): Two self mappings f and g of a fuzzy metric space (X, M, ∗) are said to satisfy the (E.A) property if there exist a sequence $\{x_n\}$ in X such that for all t > 0,

$$\lim_{n\to\infty} M(fx_n, gx_n, t) = 1.$$

Definition 2.10 (Ali et al., 2010): Two pairs (A, S) and (B, T) of self mappings of a fuzzy metric space (X, M, ∗) are said to satisfy the (E.A) property if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that for all t > 0,

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z, \text{ for some } z \text{ in } X.$$

3. Main Results:
Definition 3.1:
Implicit Relation

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Let Φ be the class of all real valued, non-decreasing and continuous functions, $\phi: (R^+)^4 \to R$, satisfying the following condition:

$\phi(x, y, x, y) \geq 0$ or $\phi(x, y, y, x) \geq 0$ or $(x, x, y, y) \geq 0$ implies $x \geq y$, for all $x, y \geq 0$;

.

Theorem 3.2:

Let A, B and < F_i > where i∈ N∪{0}, be self-maps of fuzzy metric space (X, M, ∗), satisfying the following conditions,

I. $F_i \subseteq B$ and $F_0 \subseteq A$;

II. $\phi(M(F_i x, F_0 y, kt), M(Ax, By, t), M(F_i x, Ax, t), M(F_0 y, By, kt)) \geq 0$;

III. The pairs (F_i, A) and (F_0, B) share the common property (E.A.);

IV. The pairs (F_i, A) and (F_0, B) are R-weakly commuting,

for all x, y ∈ X, t > 0, k ∈ (0, 1). If range of one of A and B is closed subspace of X, then < F_i > where i∈ N∪{0}, A and B have a unique common fixed point.

Proof: Since The pairs (F_i, A) and (F_0, B) share the common property (E.A.), there exist two sequences $\{x_n\}$ and $\{y_n\}$ such that,

$$\lim_{n\to\infty} F_0\, x_n = \lim_{n\to\infty} B\, x_n = \lim_{n\to\infty} F_i\, y_n = \lim_{n\to\infty} A\, y_n = z$$

Suppose that A(X) is a closed subspace of X, then there exists some u ∈ X such that z = Au.

Now we show that $F_i u = z$. using condition II with x = u and y = x_n.

$$\phi(M(F_i u, F_0 x_n, kt), M(Au, Bx_n, t), M(F_i u, Au, t), M(F_0 x_n, Bx_n, kt)) \geq 0$$

Taking limit as n→∞

$$\phi(M(F_i u, z, kt), M(z, z, t), M(F_i u, Au, t), M(z, z, kt)) \geq 0$$
$$\phi(M(F_i u, z, kt), 1, M(F_i u, Au, t), 1) \geq 0$$

Since ϕ is non decreasing therefore,

$$\phi(M(F_i u, z, t), 1, M(F_i u, Au, t), 1) \geq 0$$

From the definition 3.1

$$M(F_i u, z, t) \geq 1$$

Therefore, $F_i u = z = Au$.

Since $F_i \subseteq B$, there exists some v ∈ X, such that $F_i u = z = Bv$.

Now we show that $F_0 v = z$. using condition II with y = v and x = y_n.

$$\phi(M(F_i y_n, F_0 v, kt), M(Ay_n, Bv, t), M(F_i y_n, Ay_n, t), M(F_0 v, Bv, kt)) \geq 0$$

Taking limit as n→∞

$$\phi(\ M(z, F_0 v, kt), M(z, z, t), M(z, z, t), M(F_0 v, z, kt)\ ) \geq 0$$

$$\phi(\ M(z, F_0 v, kt), 1, 1, M(F_0 v, z, kt)\ ) \geq 0$$

From the definition 3.1

$$M(F_0 v, z, kt) \geq 1$$

Therefore, $F_0 v = z = Bv$,

Hence, F_iu = Au = $F_0 v = Bv = z$.

Since F_i and A are pointwise R-weakly commutting, there exists R > 0 such that,

M (F_iAu, AF_iu, t) ≥ $M(F_i \mathrm{u}, \mathrm{Au}, \frac{t}{R})$ = 1, hence F_iAu = AF_iu = F_iF_iu = AAu.

Similarily F_0 and B are pointwise R-weakly commutting, there exists R > 0 such that,

M (F_0Bv, BF_0v, t) ≥ $M(F_0 \mathrm{v}, \mathrm{Bv}, \frac{t}{R})$ = 1 hence F_0Bv = BF_0v = F_0F_0v = BBv.

using condition II with y = v and x = F_iu,

$$\phi(\ M(F_i F_i u, F_0 v, kt), M(AF_i u, Bv, t), M(F_i F_i u, AF_i u, t), M(F_0 v, Bv, kt)\ ) \geq 0$$

$$\phi(\ M(F_i F_i u, F_i u, kt), M(F_i F_i u, F_i u, t), 1, 1\ ) \geq 0$$

Since ϕ is non decreasing therefore,

$$\phi(\ M(F_i F_i u, F_i u, t), M(F_i F_i u, F_i u, t), 1, 1\ ) \geq 0$$

From the definition 3.1,

$$M(F_i F_i u, F_i u, t) \geq 1$$

It is possible only when, $F_i F_i u = F_i u$
Hence, $F_i z = z$
Therefore, $F_i z = z = Az$
Therefore z is a fixed point of F_i and A.
Now using condition II with y = F_0v and x = u,

$$\phi(\ M(F_i u, F_0 F_0 \mathrm{v}, kt), M(Au, BF_0 \mathrm{v}, t), M(F_i u, Au, t), M(F_0 F_0 \mathrm{v}, BF_0 \mathrm{v}, kt)\ ) \geq 0$$

$$\phi(\ M(F_0 v, F_0 F_0 \mathrm{v}, kt), M(F_0 v, F_0 F_0 \mathrm{v}, t), 1, 1\ ) \geq 0$$

Since ϕ is non decreasing therefore,

$$\phi(\ M(F_0 v, F_0 F_0 \mathrm{v}, t), M(F_0 v, F_0 F_0 \mathrm{v}, t), 1, 1\ ) \geq 0$$

From the definition 3.1,

$$M(F_0 v, F_0 F_0 \mathrm{v}, t) \geq 1$$

It is possible only when, $F_0 v = F_0 F_0 \mathrm{v} = \mathrm{z}$
Hence, $F_0 z = z$
Therefore, $F_0 z = Bz = z$
Therefore z is a fixed point of F_0 and B.
Which gives, $F_i z \;= A = F_0 z = Bz = z.$
Hence z is a common fixed point of $< F_i >$, A and B.

Now to show that z is unique common fixed point of $< F_i >$, where $i \in N \cup \{0\}$, A and B.
Suppose z_1 is another common fixed point of $< F_i >$, where $i \in N \cup \{0\}$, A and B.
Now using condition II with $y = z_1$ and $x = z$,

$$\phi(M(F_i z, F_0 z_1, kt), M(Az, Bz_1, t), M(F_i z, Az, t), M(F_0 z_1, Bz_1, kt)) \geq 0$$

$$\phi(M(z, z_1, kt), M(z, z_1, t), 1, 1) \geq 0$$

Since ϕ is non decreasing therefore,

$$\phi(M(z, z_1, t), M(z, z_1, t), 1, 1) \geq 0$$

From the definition 3.1,

$$M(z, z_1, t) \geq 1$$

It is possible only when, $z = z_1$
Hence z is unique common fixed point of $< F_i >$ where $i \in N \cup \{0\}$, A and B.

Theorem 3.3:

Let A, B, F and G be self-maps of fuzzy metric space (X, M, $*$), satisfying the following conditions,

I. $F \subseteq B$ and $G \subseteq A$;
II. $\phi(M(Fx, Gy, kt), M(Ax, By, t), M(Fx, Ax, \; t), M(Gy, By, kt)) \geq 0$;
III. The pairs (F, A) and (G, B) share the common property (E.A.);
IV. The pairs (F, A) and (G, B) are R-weakly commuting,

for all x, $y \in X$, $t > 0$, $k \in (0, 1)$. If range of one of A and B is closed subspace of X, then F, G, A and B have unique common fixed point.

Proof: Since The pairs (F, A) and (G, B) share the common property (E.A.), there exist two sequences $\{x_n\}$ and $\{y_n\}$ such that,

$$\lim_{n\to\infty} G\,x_n = \lim_{n\to\infty} B\,x_n = \lim_{n\to\infty} F\,y_n = \lim_{n\to\infty} A\,y_n = z$$

Suppose that A(X) is a closed subspace of X, then there exists some $u \in X$ such that $z = Au$.

Now we show that $Fu = z$. using condition II with $x = u$ and $y = x_n$.

$$\phi(M(Fu, Gx_n, kt), M(Au, Bx_n, t), M(Fu, Au, t), M(Gx_n, Bx_n, kt)) \geq 0$$

Taking limit as n→∞

$$\phi(\, M(Fu,z,kt), M(z,z,t), M(Fu,Au,t), M(z,z,kt)\,) \geq 0$$

$$\phi(\, M(Fu,z,kt), 1, M(Fu,Au,t), 1\,) \geq 0$$

Since ϕ is non decreasing therefore,

$$\phi(\, M(Fu,z,t), 1, M(Fu,Au,t), 1\,) \geq 0$$

From the definition 3.1,

$$M(Fu,z,kt) \geq 1$$

Therefore Fu = z = Au.

Since F⊆ B, there exists some v ∈ X, such that Fu = z = v.

Now we show that, Gv = z. using condition II with y = v and x = y_n.

$$\phi(\, M(Fy_n,Gv,kt), M(Ay_n,Bv,t), M(Fy_n,Ay_n,t), M(Gv,Bv,kt)\,) \geq 0$$

Taking limit as n→∞

$$\phi(\, M(z,Gv,kt), M(z,z,t), M(z,z,t), M(Gv,z,kt)\,) \geq 0$$

Since ϕ is non decreasing therefore,

$$M(Fv,z,kt) \geq 1$$

Therefore, $Gv = z = Bv$,
Hence, Fu = Au = $Gv = Bv = z$.
Since F and A are pointwise R-weakly commutting, there exists R > 0 such that,
M (FAu, AFu, t) ≥ $M(Fu, Au, \frac{t}{R})$ = 1 that is, FAu = AFu = FFu = AAu.
Similarly G and B are pointwise R-weakly commutting, there exists R > 0 such that,
M (GBv, BGv, t) ≥ $M(Gv, Bv, \frac{t}{R})$ = 1 that is, GBv = BGv = GGv = BBv.
using condition II with y = v and x = F_iu,

$$\phi(\, M(FFu,Gv,kt), M(AFu,Bv,t), M(FFu,AFu,t), M(Gv,Bv,kt)\,) \geq 0$$

$$\phi(\, M(FFu,Fu,kt), M(FFu,Fu,t), 1, 1\,) \geq 0$$

Since ϕ is non decreasing therefore,

$$\phi(\, M(FFu,Fu,t), M(FFu,Fu,t), 1, 1\,) \geq 0$$

From the definition 3.1,

$$M(FFu,Fu,t) \geq 1$$

It is possible only when, $FFu = Fu$
Hence, $Fz = z$
Therefore, $Fz = z = Az$
Therefore z is a fixed point of F and A.

Now using condition II with y = Gv and x = u,

$$\phi(M(Fu, GGv, kt), M(Au, BGv, t), M(Fu, Au, t), M(GGv, BGv, kt)) \geq 0$$

$$\phi(M(GGv, Gv, kt), M(GGv, Gv, t), 1, 1) \geq 0$$

Since ϕ is non decreasing therefore,

$$\phi(M(GGv, Gv, t), M(GGv, Gv, t), 1, 1) \geq 0$$

From the definition 3.1,

$$M(GGv, Gv, t) \geq 1$$

It is possible only when, $Gv = GGv = z$
Hence, $Gz = z$
Therefore, $Gz = Bz = z$
Therefore z is a fixed point of G and B.
Which gives, $Fz = A = Gz = Bz = z$.
Hence z is a common fixed point of F, G, A and B.

Now to show that z is unique common fixed point of F, G, A and B.

Suppose z_1 is another common fixed point of F, G, A and B.

Now using condition II with y = z_1 and x = z,

$$\phi(M(Fz, Gz_1, kt), M(Az, Bz_1, t), M(Fz, Az, t), M(Fz_1, Bz_1, kt)) \geq 0$$

$$\phi(M(z, z_1, kt), M(z, z_1, t), 1, 1) \geq 0$$

Since ϕ is non decreasing therefore,

$$\phi(M(z, z_1, t), M(z, z_1, t), 1, 1) \geq 0$$

From the definition 3.1,

$$M(z, z_1, t) \geq 1$$

It is possible only when, z = z_1.
Hence z is unique common fixed point of F, G, A and B.

Corollary 3.4:
Let A, B and < F_i > where i∈ N∪{0}, be self-maps of fuzzy metric space (X, M, ∗), satisfying the following conditions,

I. $F_i \subseteq B$ and $F_0 \subseteq A$;
II. $\phi(M(F_i x, F_0 y, kt), M(Ax, By, t), M(F_i x, Ax, t), M(F_0 y, By, kt)) \geq 0$;
III. The pair (F_0, B) satisfies property (E.A.);
IV. The pairs (F_i, A) and (F_0, B) are R-weakly commuting,

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for all x, y ∈ X, t > 0, k ∈ (0, 1). If range of one of A and B is closed subspace of X, then < F_i > where i∈ N ∪ {0}, A and T have a unique common fixed point.

Proof:

Since The pair (F_0, B) share the common property (E.A), there exist a sequence {x_n} such that,
$\lim_{n\to\infty} F_0\, x_n = \lim_{n\to\infty} B\, x_n = z$, for some z ∈ X.
Since $F_0(X) \subseteq A(X)$, there exists { y_n } in X such that, $F_0 x_n = Ay_n$ and $\lim_{n\to\infty} Ay_n = z$.
Now we show that, $\lim_{n\to\infty} F_i\, y_n = z$.
Using condition II with x = y_n and y = x_n.

$$\phi(\, M(F_i y_n, F_0 x_n, kt), M(Ay_n, Bx_n, t), M(F_i y_n, Ay_n, t), M(F_0 x_n, Bx_n, kt)\,) \geq 0$$

Taking limit as n→∞

$$\phi(\, M(F_i y_n, z, kt), M(z, z, t), M(F_i y_n, z, t), M(z, z, kt)\,) \geq 0$$

$$\phi(\, M(F_i y_n, z, kt), 1, M(F_i y_n, z, t), 1\,) \geq 0$$

Since ϕ is non decreasing therefore,

$$\phi(\, M(F_i y_n, z, t), 1, M(F_i y_n, z, t), 1\,) \geq 0$$

From the definition 3.1,

$$M(F_i y_n, z, t) \geq 1$$

It is possible only when, $\lim_{n\to\infty} F_i\, y_n = z$

Therefore,

$$\lim_{n\to\infty} F_0\, x_n = \lim_{n\to\infty} B\, x_n = \lim_{n\to\infty} F_i\, y_n = \lim_{n\to\infty} A\, y_n = z$$

The pairs (F_i, A) and (F_0, B) share the common property (E.A).

Therefore all the conditions of theorem 3.1 are satisfied hence < F_i > where i∈ N∪{0}, A and B have a unique common fixed point.

Corollary 3.5:
Let A and B be self-mappings of fuzzy metric space (X, M, ∗), satisfying the following conditions:

I. $\phi(\, M(Ax, Ay, kt), M(Bx, By, t), M(Ax, Bx,\ t), M(Ay, By, kt)\,) \geq 0$;
II. The pair (A, B) satisfies property (E.A.);
III. The pair A and B are R-weakly commuting;
IV. Range of B is closed subspace of X,

for all x, y ∈ X, t > 0, k ∈ (0, 1), then A and B have unique common fixed point in X.

Proof:
The proof can be obtained by putting $F_i = F_0$ =A and A= B in theorem 3.1.

Corollary 3.6:
Let A and I be self mappings of fuzzy metric space (X, M, ∗), satisfying the following conditions,

I. $\phi(M(Ax, Ay, kt), M(x, y, t), M(Ax, x, t), M(Ay, y, kt)) \geq 0$;
II. The pair (A, I) satisfies property (E.A.),
for all x, y ∈ X, t > 0, k ∈ (0, 1), then A has unique common fixed point in X.

Proof:
The proof can be obtained by putting $F_i = F_0 = A$ and A= B = I in theorem 3.1.

Example 3.7:
Let (X, M, ∗) be a fuzzy metric space, X = [1,-1] with $M(x, y, t) = \frac{t}{t+|x-y|}$, for all x, y ∈X, $a * b = \min\{a, b\}$ for all a, b ∈ [0, 1]., t > 0, let

$$F_i x = \frac{x}{10i} \text{ for } i \in N,\ F_0 x = 0,\ Ax = -x,\ Bx = x,\ < x_n > = \frac{1}{n},\ < y_n > = \frac{-1}{n},$$

Since,

$$\lim_{n\to\infty} F_0\, x_n = \lim_{n\to\infty} B\, x_n = \lim_{n\to\infty} F_i\, y_n = \lim_{n\to\infty} A\, y_n$$

and $Ai(X) \subseteq T(X)$, $A0(X) \subseteq S(X)$.

Also $M(F_iAx, AF_ix, t) \geq M(F_ix, Ax, \frac{t}{R})$, $M(F_0Bx, BF_0x, t) \geq M(F_ix, Sx, \frac{t}{R})$ for all x∈X,
Therefore (F_i, A) and (F_0, B) are R weakly commuting.

Let $\phi: R^+ \to R$ be defined as,

$$\phi(a, b, c, d) = a - b$$

Therefore all the conditions of theorem 3.2 are satisfied and 0 is the common fixed point.

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