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DEVELOPMENT OF FUZZY IX-MR CONTROL CHART USING FUZZY MODE AND FUZZY RULES APPROACHES

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ABSTRACT
Shewhart's control charts are the most important statistical process control tools. In statistical control charts, for definitive data, the concept of product quality is expressed by a binary function (0 and 1) to be matched or mismatched. Since a binary classification in many situations cannot be satisfactory, especially when the data are vague, fuzzy statistical control charts are introduced. The purpose of this paper is to investigate the fuzzy IX-MR control chart. Data were converted into trapezoidal fuzzy number and the fuzzy control limits were trapezoidal fuzzy number calculated using fuzzy mode and Fuzzy Rules approaches. The result was grouped into four categories (in control, out of control, rather in control and rather out of control). Finally, a case study was presented and the method coding was done in MATLAB software. The result shown that fuzzy control chart is more sensitive than classic control chart.

KEYWORDS: Fuzzy control charts, Fuzzy mode method, Fuzzy Rules method, Statistical process control, Trapezoidal fuzzy numbers.

INTRODUCTION

In the industrial world today, if products or services want to meet or exceed the customer' expectations, then they should be produced by a process that is stable. Since its products factory produced by humans and machines, there is a problem in the quality of the products. And until this problem is present, statistical methods for quality control are necessary. Among the seven tools of statistical process control, control charts, are basic and useful tools for the implementation of quality improvement programs and plays an important role in improving the quality of processes and products. And the most common is the Shewhart control charts to monitor the product or process, only is used with a qualitative characteristic. Shewhart provided the first control chart in 1926. In these charts, it is assumed that data and information are accurate and certain. Every control chart includes three parameters. The central line (CL) indicates mean values that characterize quality control processes and the upper control limit (UCL) and lower control limit (LCL) are to ensure that the process controls conditions and follow a normal distribution (Rowlands and Wang, 2000).

Most cases in industry to control the product quality characteristics are used of one sample. For example:

1. Measure the desired characteristic is done automatically.
2. Production rate is low and may not be able to analyze more than one sample.
3. The sampling process is very time consuming or expensive.
4. Tests on the prototype, ruining it.
5. Changes are not visible in the shortest time and successive measurement values are almost the same result.

In these situations the IX-MR control chart is used. In this chart, the data must be normally distributed. But when the data are equivocal and uncertain, it will lose its effectiveness charts. Fuzzy logic is used in these situations. In statistical control charts, for definitive data, the concept of product quality is expressed by a binary function (0 and 1) to be matched or mismatched. When the numbers of quality levels are between normal and defective, fuzzy statistical control charts are introduced. Wang and Raz (1990) and Raz and Wang (1990) were the first words of the language as very good, good, moderate, poor and bad accepted to express moderate levels of a qualitative character. Kanagawa *et al.* (1993) created control charts based on probability density functions as linguistic variables and established a method for monitoring the process mean and variance that was ignored by Wang and Raz (1990). Also (Kanagawa, 1993) used linguistic variables to express the process output. Then (Wang and Chen, 1995) were introduced a fuzzy mathematical programming model to design innovative economic statistical control charts. Also (Kahraman *et al.*, 1995) used of triangular fuzzy numbers in the design of control charts for linguistic data. Taleb and Limam (2002) explained another view about the construction of fuzzy control charts based on linguistic data and deduced that fuzzy control charts were

Trends in Life Sciences
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sensitive to the degree of fuzziness, which contradicted the conclusion of Wang and Raz (1990). Gullbay *et al.*, 2004 developed fuzzy control chart using a-cut method. Taleb and Limam (2005) also developed polynomial fuzzy control charts. Gullbay and Kahraman (2007) provided fuzzy control charts for linguistic variables. Senturk and Erginel (2009) designed fuzzy control charts of Xbar-R and Xbar-S using a-cut. In 2011, Xbar-R control chart was developed using fuzzy mode and fuzzy rules methods. Avakh Darestani *et al.*, (2014) developed fuzzy U control chart for monitoring defects. As mentioned, according to the real-world data and information are vague and imprecise expression, this study with development IX-MR control chart, trying to overcome the problem of inaccurate and ambiguous information. Thus, according to the study (Kaya and Kahraman, 2011) the main objective of this study was to calculate the fuzzy IX- MR control chart, who are more sensitive than other models.

MATERIALS AND METHODS

After selecting a desired characteristic, if this is not accurate, this value can be determined by trapezoidal fuzzy numbers as shown below (Kaya and Kahraman, 2011).

$$\tilde{X}_i = (X_a, X_b, X_c, X_d) \tag{1}$$

$\tilde{\bar{X}}$, MR and $\overline{\tilde{MR}}$ are calculated in Equations 2, 3 and 4.

$$\tilde{\bar{X}} = \left(\frac{\sum_{i=1}^{m} X_{ai}}{m}, \frac{\sum_{i=1}^{m} X_{bi}}{m}, \frac{\sum_{i=1}^{m} X_{ci}}{m}, \frac{\sum_{i=1}^{m} X_{di}}{m}\right) = (\bar{X}_a, \bar{X}_b, \bar{X}_c, \bar{X}_d) \tag{2}$$

$$\tilde{MR}_i = |(X_{ai}, X_{bi}, X_{ci}, X_{di}) - (X_{ai-1}, X_{bi-1}, X_{ci-1}, X_{di-1})| = |(X_{ai} - X_{di-1}, X_{bi} - X_{ci-1}, X_{ci} - X_{bi-1}, X_{di} - X_{ai-1})|$$
$$\Rightarrow \tilde{MR}_i = (MR_{ai}, MR_{bi}, MR_{ci}, MR_{di}) \tag{3}$$

$$\overline{\tilde{MR}} = \frac{\sum_{i=2}^{m-1} \tilde{MR}_i}{m-1} = \frac{\sum_{i=2}^{m-1} (MR_{ai}, MR_{bi}, MR_{ci}, MR_{di})}{m-1} = \left(\frac{\sum_{i=2}^{m-1} MR_{ai}}{m-1}, \frac{\sum_{i=2}^{m-1} MR_{bi}}{m-1}, \frac{\sum_{i=2}^{m-1} MR_{ci}}{m-1}, \frac{\sum_{i=2}^{m-1} MR_{di}}{m-1}\right)$$
$$= (\overline{MR}_a, \overline{MR}_b, \overline{MR}_c, \overline{MR}_d) \tag{4}$$

Control limits for the IX-MR chart are calculated in Equations 5- 8.

$$U\tilde{C}L_{IX} = \tilde{C}L + 3\frac{\overline{\tilde{MR}}}{d_2} = (\bar{X}_a, \bar{X}_b, \bar{X}_c, \bar{X}_d) + 3\frac{(\overline{MR}_a, \overline{MR}_b, \overline{MR}_c, \overline{MR}_d)}{d_2}$$
$$= \left(\bar{X}_a + \frac{3\overline{MR}_a}{d_2}, \bar{X}_b + \frac{3\overline{MR}_b}{d_2}, \bar{X}_c + \frac{3\overline{MR}_c}{d_2}, \bar{X}_d + \frac{3\overline{MR}_d}{d_2}\right) = (UCL_1, UCL_2, UCL_3, UCL_4) \tag{5}$$

$$\tilde{C}L_{IX} = (\bar{X}_a, \bar{X}_b, \bar{X}_c, \bar{X}_d) = (CL_1, CL_2, CL_3, CL_4) \tag{6}$$

$$L\tilde{C}L_{IX} = \tilde{C}L - 3\frac{\overline{\tilde{MR}}}{d_2} = (\bar{X}_a, \bar{X}_b, \bar{X}_c, \bar{X}_d) - 3\frac{(\overline{MR}_a, \overline{MR}_b, \overline{MR}_c, \overline{MR}_d)}{d_2}$$
$$= \left(\bar{X}_a - \frac{3\overline{MR}_d}{d_2}, \bar{X}_b - \frac{3\overline{MR}_c}{d_2}, \bar{X}_c - \frac{3\overline{MR}_b}{d_2}, \bar{X}_d - \frac{3\overline{MR}_a}{d_2}\right) = (LCL_1, LCL_2, LCL_3, LCL_4) \tag{7}$$

Trends in Life Sciences
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ISSN: 2319–4731 (p); 2319–5037 (e)

$$\begin{cases} \tilde{UCL} = D_4 \overline{\tilde{MR}} = D_4(\overline{MR}_a, \overline{MR}_b, \overline{MR}_c, \overline{MR}_d) = (UCL_1, UCL_2, UCL_3, UCL_4) \\ \tilde{CL} = \overline{\tilde{MR}} = (\overline{MR}_a, \overline{MR}_b, \overline{MR}_c, \overline{MR}_d) = (CL_1, CL_2, CL_3, CL_4) \\ \tilde{LCL} = D_3 \overline{\tilde{MR}} = D_3(\overline{MR}_a, \overline{MR}_b, \overline{MR}_c, \overline{MR}_d) = (LCL_1, LCL_2, LCL_3, LCL_4) \end{cases} \quad (8)$$

However, to determine the position of process, we used two methods fuzzy mode and fuzzy rules**.**

Fuzzy mode method

The fuzzy mode of a fuzzy set (f_{mod}) is the value of base variable where the membership function equals 1. This is stated in Equation 9 (Gulby and Kahraman, 2006, 2007)**.**

$$f_{mod} = \{x \in X \mid \mu_f(x) = 1\} \quad (9)$$

Fuzzy mode for trapezoidal numbers is shown in Figure 1.

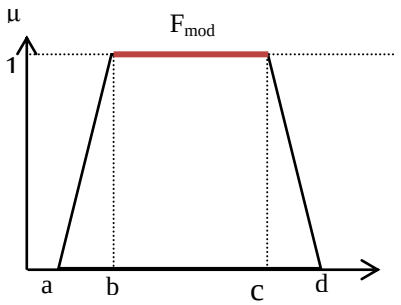

Figure 1. Fuzzy mode for trapezoidal fuzzy numbers

Control limits are calculated using fuzzy mode for trapezoidal fuzzy numbers in Equations 10 and 11.

$$Mode_{IX} = \begin{cases} UCL_{IX\,mod1} = UCL_{IX2} \\ UCL_{IX\,mod2} = UCL_{IX3} \\ CL_{IX\,mod1} = CL_{IX2} \\ CL_{IX\,mod2} = CL_{IX3} \\ LCL_{IX\,mod1} = LCL_{IX2} \\ LCL_{IX\,mod2} = LCL_{IX3} \end{cases} \quad (10)$$

$$Mode_{MR} = \begin{cases} UCL_{MR\,mod1} = UCL_{MR2} \\ UCL_{MR\,mod2} = UCL_{MR3} \\ CL_{MR\,mod1} = CL_{MR2} \\ CL_{MR\,mod2} = CL_{MR3} \\ LCL_{MR\,mod1} = LCL_{MR2} \\ LCL_{MR\,mod2} = LCL_{MR3} \end{cases} \quad (11)$$

For each sample, the fuzzy mode is calculated according to the Equation 12**.**

Trends in Life Sciences
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$$IX\ \mathrm{mod}_{i1} = X_{bi} \qquad i = 1, 2, ..., m$$
$$IX\ \mathrm{mod}_{i2} = X_{ci} \qquad i = 1, 2, ..., m$$
$$MR\ \mathrm{mod}_{i1} = MR_{bi} \qquad i = 1, 2, ..., m$$
$$MR\ \mathrm{mod}_{i2} = MR_{ci} \qquad i = 1, 2, ..., m \tag{12}$$

Respectively, indexes $(C\ IX_i)$ and $(C\ MR_i)$ for IX - MR chart are defined by the Equations 13 and 14 (Kaya and Kahraman, 2011).

$$C\ IX_i = \begin{cases} 0 & \text{if } (IX\ \mathrm{mod}_{i1} > UCL_{IX\ \mathrm{mod}2}) \\ \dfrac{UCL_{IX\ \mathrm{mod}2} - IX\ \mathrm{mod}_{i1}}{IX\ \mathrm{mod}_{i2} - IX\ \mathrm{mod}_{i1}} & \text{if } (LCL_{IX\ \mathrm{mod}1} \le IX\ \mathrm{mod}_{i1} \le UCL_{IX\ \mathrm{mod}2}) \wedge (IX\ \mathrm{mod}_{i2} \ge UCL_{IX\ \mathrm{mod}2}) \\ 1 & \text{if } (LCL_{IX\ \mathrm{mod}1} \le IX\ \mathrm{mod}_{i1}) \wedge (IX\ \mathrm{mod}_{i2} \le UCL_{IX\ \mathrm{mod}2}) \\ \dfrac{LCL_{IX\ \mathrm{mod}1} - IX\ \mathrm{mod}_{i1}}{IX\ \mathrm{mod}_{i2} - IX\ \mathrm{mod}_{i1}} & \text{if } (IX\ \mathrm{mod}_{i1} \le LCL_{IX\ \mathrm{mod}1}) \wedge (LCL_{IX\ \mathrm{mod}1} \le IX\ \mathrm{mod}_{i2} \le UCL_{IX\ \mathrm{mod}2}) \\ 0 & \text{if } (IX\ \mathrm{mod}_{i2} > LCL_{IX\ \mathrm{mod}1}) \end{cases} \tag{13}$$

$$C\ MR_i = \begin{cases} 0 & \text{if } (MR\ \mathrm{mod}_{i1} > UCL_{MR\ \mathrm{mod}2}) \\ \dfrac{UCL_{MR\ \mathrm{mod}2} - MR\ \mathrm{mod}_{i1}}{MR\ \mathrm{mod}_{i2} - MR\ \mathrm{mod}_{i1}} & \text{if } (LCL_{MR\ \mathrm{mod}1} \le MR\ \mathrm{mod}_{i1} \le UCL_{MR\ \mathrm{mod}2}) \wedge (MR\ \mathrm{mod}_{i2} \ge UCL_{MR\ \mathrm{mod}2}) \\ 1 & \text{if } (LCL_{MR\ \mathrm{mod}1} \le MR\ \mathrm{mod}_{i1}) \wedge (MR\ \mathrm{mod}_{i2} \le UCL_{IX\ \mathrm{mod}2}) \\ \dfrac{LCL_{MR\ \mathrm{mod}1} - MR\ \mathrm{mod}_{i1}}{MR\ \mathrm{mod}_{i2} - MR\ \mathrm{mod}_{i1}} & \text{if } (MR\ \mathrm{mod}_{i1} \le LCL_{MR\ \mathrm{mod}1}) \wedge (LCL_{MR\ \mathrm{mod}1} \le MR\ \mathrm{mod}_{i2} \le UCL_{MR\ \mathrm{mod}2}) \\ 0 & \text{if } (MR\ \mathrm{mod}_{i2} > LCL_{MR\ \mathrm{mod}1}) \end{cases} \tag{14}$$

And finally, the decision process is as follows (Kaya and Kahraman, 2011).

$$\text{process control} \begin{cases} \text{"in control"} & \text{if } (C\ IX_i = 1) \wedge (C\ MR_i = 1) \\ \text{"out of control"} & \text{if } (C\ IX_i = 0) \vee (C\ MR_i = 0) \\ \text{"rather in control"} & \text{if } (C\ IX_i \ge \beta) \wedge (C\ MR_i \ge \beta) \\ \text{"rather out of control"} & \text{if } (C\ IX_i < \beta) \vee (C\ MR_i < \beta) \end{cases} \tag{15}$$

Fuzzy Rules Method

Fuzzy rules introduced by Kaya and Kahraman in 2011. In the section, Fuzzy rules for trapezoidal fuzzy numbers are expressed.

Rule-1 takes into account the case where a sample mean (or a range) is between control limits. Rule-2 takes into account the case where a sample mean (or a range) is out of control limits. Rules 3 and 4 interpreted the case where a sample mean (or a range) is partially included by either of control limits. In this case, some linguistic decisions such as ‘‘rather in control” or ‘‘rather out-of control” are established. If the percentage area of a sample mean (or a range) which stays inside the fuzzy control limits is equal or greater than a predefined acceptable percentage (β), then the process can be identified as ‘‘rather in control”; if not it can be explained as ‘‘rather out of control”. Rule-5 interpreted the case where a sample mean (or a range) is partially included both of control limits (Kaya and Kahraman, 2011). These rules are shown in the following figures 2-5.

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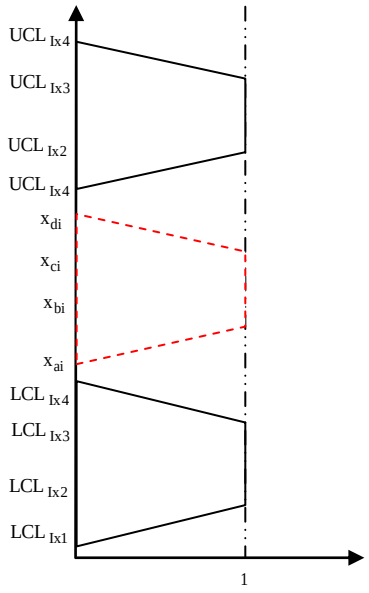

Figure 2: Fuzzy rule-1

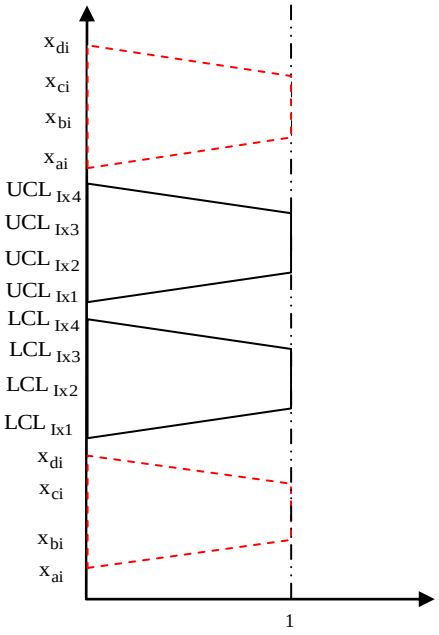

Figure 3: Fuzzy rule-2

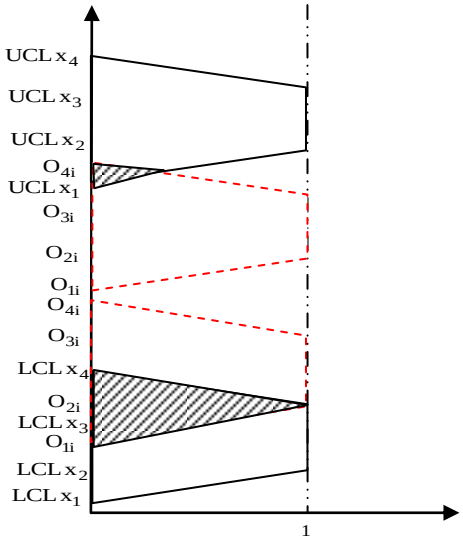

Figure 4: Fuzzy rules 3 and 4

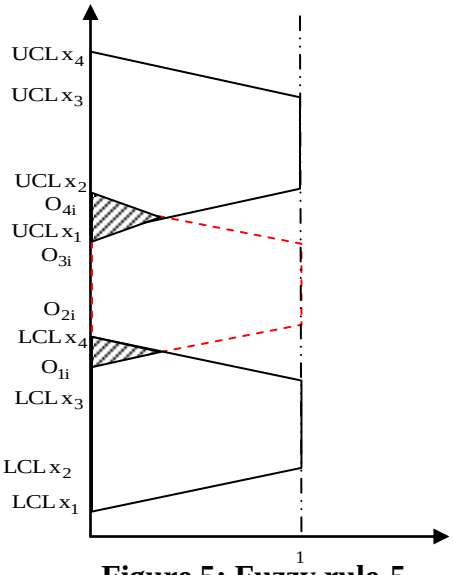

Figure 5: Fuzzy rule-5

Then for IX chart, index (C IXi) and, for MR chart, index (C MRi) are calculated by the following Equations 16 and 17 (Kaya and Kahraman, 2011).

Trends in Life Sciences
An International Peer-reviewed Journal
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$$C\ IX_i = \begin{cases} 1 & \text{if } (x_{d_i} \le UCL_{\ IX1}) \wedge (x_{a_i} \ge LCL_{IX4}) \\ 0 & \text{if } (x_{a_i} > UCL_{IX4}) \vee (x_{d_i} < LCL_{IX1}) \\ 1-\dfrac{(x_{d_i} - UCL_{IX_i})}{(x_{d_i} - x_{a_i})} & \text{if } (x_{d_i} > UCL_{IX1}) \\ 1-\dfrac{(LCL_{IX_3} - x_{a_i})}{(x_{d_i} - x_{a_i})} & \text{if } (x_{a_i} < LCL_{IX4}) \\ \text{Min}\left\{1-\dfrac{(x_{d_i} - UCL_{IX_1})}{(x_{d_i} - x_{a_i})}, 1-\dfrac{(LCL_{IX_3} - x_{a_i})}{(x_{d_i} - x_{a_i})}\right\} & \text{if } (x_{d_i} > UCL_{IX1}) \wedge (x_{a_i} < LCL_{IX4}) \end{cases} \tag{16}$$

$$C\ MR_i = \begin{cases} 1 & \text{if } (MR_{d_i} \le UCL_{\ MR1}) \wedge (MR_{ai} \ge LCL_{MR4}) \\ 0 & \text{if } (MR_{a_i} > UCL_{MR4}) \vee (MR_{di} < LCL_{MR1}) \\ 1-\dfrac{(MR_{d_i} - UCL_{MR_1})}{(MR_{d_i} - MR_{a_i})} & \text{if } (MR_{d_i} > UCL_{MR1}) \\ 1-\dfrac{(LCL_{MR_4} - MR_{a_i})}{(MR_{d_i} - MR_{a_i})} & \text{if } (MR_{a_i} < LCL_{MR4}) \\ \text{Min}\left\{1-\dfrac{(MR_{d_i} - UCL_{MR_1})}{(MR_{d_i} - MR_{a_i})}, 1-\dfrac{(LCL_{MR_4} - MR_{a_i})}{(MR_{d_i} - MR_{a_i})}\right\} & \text{if } (MR_{d_i} > UCL_{MR1}) \wedge (MR_{a_i} < LCL_{MR4}) \end{cases} \tag{17}$$

And finally, the decision process is as follows (Kaya and Kahraman, 2011).

$$\text{process control} \begin{cases} \text{"in control"} & \text{if } (C_{IXi} = 1) \wedge (C_{MRi} = 1) \\ \text{"out of control"} & \text{if } (C_{IXi} = 0) \vee (C_{MRi} = 0) \\ \text{"rather in control"} & \text{if } (C_{IXi} \ge \beta) \wedge (C_{MRi} \ge \beta) \\ \text{"rather out of control"} & \text{if } (C_{IXi} < \beta) \vee (C_{MRi} < \beta) \end{cases} \tag{18}$$

RESULTS AND DISCUSSION

In this section, in order to illustrate the proposed model, we solved a numerical example with fuzzy mode and fuzzy rules methods. The data consisted of 125 samples which are normally distributed. Normality test was performed by the software Minitab and is shown in Figure 6. According to Figure 6, the data on one line and the p-value is greater than 0.05, which indicates that the data are normally distributed. Linguistic terms with trapezoidal fuzzy numbers are r shown in Table 1.

Trends in Life Sciences
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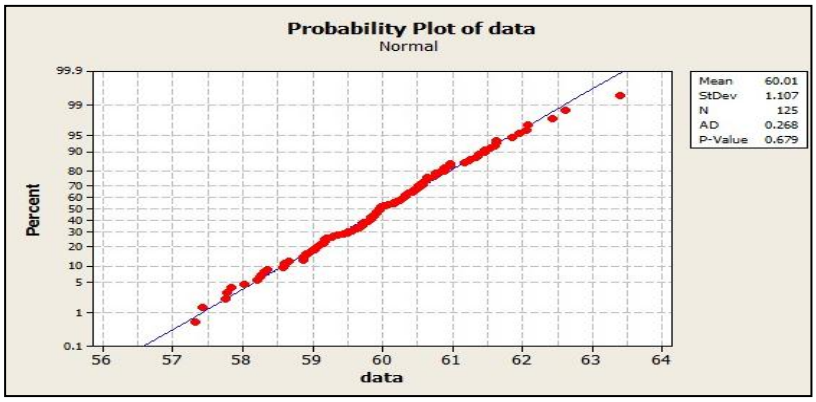

Figure 6: normality test using MINITAB software

Table 1. Trapezoidal fuzzy numbers

NO	Trapezoidal fuzzy numbers				NO	Trapezoidal fuzzy numbers			
	a	b	c	d		a	b	c	d
1	59.175	59.176	59.178	59.179	32	60.956	60.957	60.959	60.96
2	60.925	60.926	60.928	60.929	33	59.277	59.278	59.28	59.281
3	58.66	58.661	58.663	58.664	34	59.091	59.092	59.094	59.095
4	59.677	59.678	59.68	59.681	35	62.07	62.071	62.073	62.074
5	61.233	61.234	61.236	61.237	36	59.664	59.665	59.667	59.668
6	59.571	59.572	59.574	59.575	37	59.795	59.796	59.798	59.799
7	59.122	59.123	59.125	59.126	38	60.627	60.628	60.63	60.631
8	59.717	59.718	59.72	59.721	39	61.452	61.453	61.455	61.456
9	60.24	60.241	60.243	60.244	40	59.868	59.869	59.871	59.872
10	59.513	59.514	59.516	59.517	41	61.624	61.625	61.627	61.628
11	59.595	59.596	59.598	59.599	42	61.173	61.174	61.176	61.177
12	60.377	60.378	60.38	60.381	43	60.431	60.432	60.434	60.435
13	59.982	59.983	59.985	59.986	44	60.459	60.46	60.462	60.463
14	61.955	61.956	61.958	61.959	45	60.16	60.161	60.163	60.164
15	60.038	60.039	60.041	60.042	46	60.202	60.203	60.205	60.206
16	57.768	57.769	57.771	57.772	47	61.371	61.372	61.374	61.375
17	60.767	60.768	60.77	60.771	48	59.826	59.827	59.829	59.83
18	60.332	60.333	60.335	60.336	49	60.885	60.886	60.888	60.889
19	61.326	61.327	61.329	61.33	50	60.873	60.874	60.876	60.877
20	58.913	58.914	58.916	58.917	51	60.597	60.598	60.6	60.601
21	59.056	59.057	59.059	59.06	52	60.816	60.817	60.819	60.82
22	57.839	57.84	57.842	57.843	53	58.865	58.866	58.868	58.869
23	58.994	58.995	58.997	58.998	54	58.583	58.584	58.586	58.587
24	60.621	60.622	60.624	60.625	55	61.461	61.462	61.464	61.465
25	59.172	59.173	59.175	59.176	56	61.366	61.367	61.369	61.37
26	58.977	58.978	58.98	58.981	57	59.82	59.821	59.823	59.824
27	57.756	57.757	57.759	57.76	58	59.917	59.918	59.92	59.921
28	59.835	59.836	59.838	59.839	59	60.571	60.572	60.574	60.575
29	60.645	60.646	60.648	60.649	60	59.958	59.959	59.961	59.962
30	60.739	60.74	60.742	60.743	61	60.29	60.291	60.293	60.294
31	59.838	59.839	59.841	59.842	62	60.58	60.581	60.583	60.584

Trends in Life Sciences
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Table 1. Trapezoidal fuzzy numbers

NO	Trapezoidal fuzzy numbers				NO	Trapezoidal fuzzy numbers			
	a	b	c	d		a	b	c	d
63	58.901	58.902	58.904	58.905	95	60.646	60.647	60.649	60.65
64	58.021	58.022	58.024	58.025	96	61.244	61.245	61.247	61.248
65	59.964	59.965	59.967	59.968	97	60.509	60.51	60.512	60.513
66	59.932	59.933	59.935	59.936	98	57.424	57.425	57.427	57.428
67	58.265	58.266	58.268	58.269	99	58.257	58.258	58.26	58.261
68	59.433	59.434	59.436	59.437	100	60.584	60.585	60.587	60.588
69	58.875	58.876	58.878	58.879	101	57.328	57.329	57.331	57.332
70	60.962	60.963	60.965	60.966	102	60.453	60.454	60.456	60.457
71	60.08	60.081	60.083	60.084	103	59.035	59.036	59.038	59.039
72	58.318	58.319	58.321	58.322	104	60.269	60.27	60.272	60.273
73	61.859	61.86	61.862	61.863	105	59.067	59.068	59.07	59.071
74	59.16	59.161	59.163	59.164	106	59.871	59.872	59.874	59.875
75	60.155	60.156	60.158	60.159	107	60.317	60.318	60.32	60.321
76	58.346	58.347	58.349	58.35	108	62.049	62.05	62.052	62.053
77	59.686	59.687	59.689	59.69	109	59.626	59.627	59.629	59.63
78	60.465	60.466	60.468	60.469	110	59.728	59.729	59.731	59.732
79	59.826	59.827	59.829	59.83	111	62.43	62.431	62.433	62.434
80	60.761	60.762	60.764	60.765	112	60.521	60.522	60.524	60.525
81	59.91	59.911	59.913	59.914	113	59.735	59.736	59.738	59.739
82	59.192	59.193	59.195	59.196	114	62.621	62.622	62.624	62.625
83	60.506	60.507	60.509	60.51	115	59.717	59.718	59.72	59.721
84	60.89	60.891	60.893	60.894	116	60.267	60.268	60.27	60.271
85	58.207	58.208	58.21	58.211	117	59.301	59.302	59.304	59.305
86	61.624	61.625	61.627	61.628	118	60.017	60.018	60.02	60.021
87	60.437	60.438	60.44	60.441	119	58.603	58.604	58.606	58.607
88	59.513	59.514	59.516	59.517	120	59.978	59.979	59.981	59.982
89	59.359	59.36	59.362	59.363	121	58.593	58.594	58.596	58.597
90	59.925	59.926	59.928	59.929	122	61.606	61.607	61.609	61.61
91	58.871	58.872	58.874	58.875	123	59.895	59.896	59.898	59.899
92	63.388	63.389	63.391	63.392	124	60.329	60.33	60.332	60.333
93	59.165	59.166	59.168	59.169	125	60.348	60.349	60.351	60.352
94	61.534	61.535	61.537	61.538					

In this section, trapezoidal fuzzy control limits are calculated for IX-MR control chart. Modeling of the control chart was coded by MATLAB software. Control limits are calculated as shown in Table 2.

Table 2. Fuzzy IX-MR control limit

IX control chart	
UCL	(63.4390, 63.4459, 63.4579, 63.4643)
CL	(60.0071, 60.0081, 60.0101, 60.0111)
LCL	(56.5539, 56.5602, 56.5729, 56.5792)
MR control chart	
UCL	(4.2157, 4.2223, 4.2353, 4.2419)
CL	(1.2904, 1.2924, 1.2964, 1.2984)
LCL	(0,0,0,0)

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After calculated the trapezoidal fuzzy control limits, we must evaluate the process. For this purpose, we used fuzzy mode and fuzzy rules method. The result was grouped into four categories (in control, out of control, rather in control, rather out of control). Then, the results respectively were given in Tables 3 and 4. According to Table 3, by using Fuzzy mode method, data 92 was "out of control". Also, according to Table 4, by using Fuzzy rules method, data 92 was "out of control" and data 93 was "rather out of control". Therefore, these data should be deleted and re-calculate fuzzy control limits for these data. New control limits are shown in Table 5.

Table 3. Results of the process control by using Fuzzy mode method

NO	IX	MR	NO	IX	MR	NO	IX	MR
1	in control		43	in control	in control	85	in control	in control
2	in control	in control	44	in control	in control	86	in control	in control
3	in control	in control	45	in control	in control	87	in control	in control
4	in control	in control	46	in control	in control	88	in control	in control
5	in control	in control	47	in control	in control	89	in control	in control
6	in control	in control	48	in control	in control	90	in control	in control
7	in control	in control	49	in control	in control	91	in control	in control
8	in control	in control	50	in control	in control	92	in control	Out of control
9	in control	in control	51	in control	in control	93	in control	in control
10	in control	in control	52	in control	in control	94	in control	in control
11	in control	in control	53	in control	in control	95	in control	in control
12	in control	in control	54	in control	in control	96	in control	in control
13	in control	in control	55	in control	in control	97	in control	in control
14	in control	in control	56	in control	in control	98	in control	in control
15	in control	in control	57	in control	in control	99	in control	in control
16	in control	in control	58	in control	in control	100	in control	in control
14	in control	in control	59	in control	in control	101	in control	in control
18	in control	in control	60	in control	in control	102	in control	in control
19	in control	in control	61	in control	in control	103	in control	in control
20	in control	in control	62	in control	in control	104	in control	in control
21	in control	in control	63	in control	in control	105	in control	in control
22	in control	in control	64	in control	in control	106	in control	in control
23	in control	in control	65	in control	in control	107	in control	in control
24	in control	in control	66	in control	in control	108	in control	in control
25	in control	in control	67	in control	in control	109	in control	in control
26	in control	in control	68	in control	in control	110	in control	in control
27	in control	in control	69	in control	in control	111	in control	in control
28	in control	in control	70	in control	in control	112	in control	in control
29	in control	in control	71	in control	in control	113	in control	in control
30	in control	in control	72	in control	in control	114	in control	in control
31	in control	in control	73	in control	in control	115	in control	in control
32	in control	in control	74	in control	in control	116	in control	in control
33	in control	in control	75	in control	in control	117	in control	in control
34	in control	in control	76	in control	in control	118	in control	in control
35	in control	in control	77	in control	in control	119	in control	in control
36	in control	in control	78	in control	in control	120	in control	in control
37	in control	in control	79	in control	in control	121	in control	in control
38	in control	in control	80	in control	in control	122	in control	in control
39	in control	in control	81	in control	in control	123	in control	in control
40	in control	in control	82	in control	in control	124	in control	in control
41	in control	in control	83	in control	in control	125	in control	in control
42	in control	in control	84	in control	in control			

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Table 4. Results of the process control by using Fuzzy rules method

NO	IX	MR	NO	IX	MR	NO	IX	MR
1	in control		43	in control	in control	85	in control	in control
2	in control	in control	44	in control	in control	86	in control	in control
3	in control	in control	45	in control	in control	87	in control	in control
4	in control	in control	46	in control	in control	88	in control	in control
5	in control	in control	47	in control	in control	89	in control	in control
6	in control	in control	48	in control	in control	90	in control	in control
7	in control	in control	49	in control	in control	91	in control	in control
8	in control	in control	50	in control	in control	92	in control	Out of control
9	in control	in control	51	in control	in control	93	in control	Rather out of control
10	in control	in control	52	in control	in control	94	in control	in control
11	in control	in control	53	in control	in control	95	in control	in control
12	in control	in control	54	in control	in control	96	in control	in control
13	in control	in control	55	in control	in control	97	in control	in control
14	in control	in control	56	in control	in control	98	in control	in control
15	in control	in control	57	in control	in control	99	in control	in control
16	in control	in control	58	in control	in control	100	in control	in control
14	in control	in control	59	in control	in control	101	in control	in control
18	in control	in control	60	in control	in control	102	in control	in control
19	in control	in control	61	in control	in control	103	in control	in control
20	in control	in control	62	in control	in control	104	in control	in control
21	in control	in control	63	in control	in control	105	in control	in control
22	in control	in control	64	in control	in control	106	in control	in control
23	in control	in control	65	in control	in control	107	in control	in control
24	in control	in control	66	in control	in control	108	in control	in control
25	in control	in control	67	in control	in control	109	in control	in control
26	in control	in control	68	in control	in control	110	in control	in control
27	in control	in control	69	in control	in control	111	in control	in control
28	in control	in control	70	in control	in control	112	in control	in control
29	in control	in control	71	in control	in control	113	in control	in control
30	in control	in control	72	in control	in control	114	in control	in control
31	in control	in control	73	in control	in control	115	in control	in control
32	in control	in control	74	in control	in control	116	in control	in control
33	in control	in control	75	in control	in control	117	in control	in control
34	in control	in control	76	in control	in control	118	in control	in control
35	in control	in control	77	in control	in control	119	in control	in control
36	in control	in control	78	in control	in control	120	in control	in control
37	in control	in control	79	in control	in control	121	in control	in control
38	in control	in control	80	in control	in control	122	in control	in control
39	in control	in control	81	in control	in control	123	in control	in control
40	in control	in control	82	in control	in control	124	in control	in control
41	in control	in control	83	in control	in control	125	in control	in control
42	in control	in control	84	in control	in control			

Table 5. New fuzzy IX-MR control limit

IX control chart	
UCL	(63.3153, 63.3217, 63.3343, 63.3406)
CL	(59.9837, 59.9847, 59.9867, 59.9877)
LCL	(56.6308, 56.6372, 56.6498, 56.6516)
MR control chart	
UCL	(4.0925, 4.0991, 4.1121, 4.1187)
CL	(1.2527, 1.2547, 1.2587, 1.2607)
LCL	(0,0,0,0)

RESULTS AND DISCUSSION

In this research, we developed fuzzy IX-MR control chart, and we decision-making about process by two approaches. The proposed model was coded in MATLAB. Classical control charts were defined in two groups of in- control and out-of-control. The results show that the proposed model was more sensitive than Classical control charts and the process into four categories, in control, out of control, rather in control and rather out of control. After analyzing the data, it was concluded that fuzzy rules method was more sensitive than fuzzy mode in number 93 and the rest of the results obtained by both methods were identical. For future research, capability indices for IX-MR chart can be recommended in the fuzzy environment.

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