

## PRESENTING OF FUZZY STATISTICAL PROCESS CONTROL USING FUZZY MIDDLE LIMIT METHOD AND DIRECT FUZZY APPROACH FOR CONTROLLING THE NUMBER OF DEFICIENCIES IN A PRODUCT

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### ABSTRACT

Classic Control Diagrams Classify Production Processes into two groups: under control or out of control, Uses exact and definite data. But Fuzzy control diagrams are used to control the process when the data are ambiguous and non-meticulous. These diagrams divide products into groups “under control, relatively under control, relatively out of control and out of control”. Fuzzy logic presents a regular base for encountering ambiguous or unclear condition. Using the fuzzy methods in the early stage of algorithm makes it resemble classic analysis more than usual. In this article in addition to non-definite and individuals’ mental experience, multi-level quality of the product in the form of “under control, vocatively under control, relatively out of control and out of control” are defined using fuzzy middle limit with  $\alpha$  level and  $a$ -cut middle fuzzy through drawing fuzzy control diagrams. Considering the point that in the fuzzy methods, the data are partly lost, thus direct fuzzy approach has been used.

**KEYWORDS:** A-cut, De fuzzy, Direct fuzzy approach, Fuzzy control Diagram.

### INTRODUCTION:

Statistical control diagrams that were used in 1920 for the first time are common now, too. But in some cases and condition where existing data have ambiguity and are not certain, they lose their efficiency. To empower statistical controls, strategies have been recommended that the most important of which is the use of fuzzy logic in statistical control. Fuzzy quality control which is in its early steps, given the existing capability in formulating the experience of experts and use of ambiguous and non-meticulous, this raises the quality control toward its main mission that is control inspection and improving products and service quality. Product control diagrams are put into two categories of zero and one in two groups of under control or out of safe control or deficient control or control or conforming or none conforming. This type of classification in the condition that the quality of the product does not change suddenly from satisfactory to dissatisfactory situation will not be appropriate. In classic division, if the selected sample is very close to upper and lower limit controls but inside control limit, it is called conforming product and if it is close to lower and upper control. In case, other classifications in the form of intermediate ones between conforming and none conforming or under control or out of control are considered, the quality level of the product will be stated in a more real way. Fuzzy control diagrams study the performance of processes with fuzzy data. In the theory of fuzzy sets, ambiguous data are considered in the form of fuzzy sets, then, each set with non-fuzzy factors such as mean, mode and fuzzy mode are converted into clear and exact figures and then control diagrams are created. One disadvantage of these methods is that first of all the data of fuzzy sets are lost due to using non-fuzzy factors. In this article, different sections are organized as following: in section two, the literature review, in section three, the study of the subject with details and in section four, the study of the subject in fuzzy state with fuzzy middle limit method and direct fuzzy approach (DFA) are covered. In section five, the analysis of the subject in two states, that is, exact and fuzzy state is conducted and then the conclusion and recommendations are presented. In the final section, the references are stated (Ismaelpur *et al.* 2009).

Numerous studies have been conducted on using statistical fuzzy control methods including fuzzy set theory mentioned by (shaw, 1985) for the first time, as a base for interpreting the presentation of grading performance degree. (Raz and wong, 1990) and (Wong and raz, 1990), two separate articles, used approaches for overcoming the limitations of classification used in descriptive control diagrams in order to mention the internal level of the quality more and better than classic terms but their approach is different from that of Raz and Wang since they have used the probability density function for studying the probability of observing linguistic variables (Kahraman and colleagues, 1995) used triangle fuzzy figures for testing control diagrams for abnormal data. Through creating a fuzzy concluding system, (El-

Shal and Moris, 2000) raised the deficiency discovery rate in production line. Methods for building the diagram of linguistic data control based on probability theory and fuzzy sets were discussed by (Taleb and limam.2002).

Golby and colleagues (2004) recommended diagrams introduced by (Wong and Raz, 1990) using the concept of alpha level for descriptive diagrams. It was concluded that fuzzy control diagrams reveal the quality level and the rate of control with more accuracy. The purpose of using alpha the exact formulating of inspection behavior and existing ambiguity in fuzzy diagram. (Cheng, 2005) suggested an alternative approach for using expert mental judgment. In this approach fuzzy figure (numbers) are made using samples ranked by different experts and relation matrices.

(Galbay and khahraman,2006) introduced fuzzy rules identifying unusual patterns.(Ismaelpur and colleagues, 2009) presented a model for controlling the number of deficiencies of the product using fuzzy mode compared to classic method.

### To study the subject in detail:

Statistical control process (SPC) is a systematic method for solving the problem or improving the process and controlling processes in a way that the output quality of the process becomes sure. To solve each type of problem it is required that steps such as following ones be taken and statistical control process helps the efficient performance of each step by presenting seven quality tools as follows: data record sheet , deficiency concentration diagram, histogram, distribution , parto diagram, control diagram and cause and effect diagram. Control diagram is of the most important tools. In these diagrams, the process finctuations could be controlled. The selection of control diagrams is not based on the data type and the sample volume under control or natural state of the production process, but it is an achievement that must be detained through constant attempt and removing the individual causes of numerous variables.

Control diagrams are used for quality and quantity samples. Quantity diagrams consist of R, X, S and I-R diagrams and there are four diagrams for descriptive detail the deficient proportionate diagram (P), deficient number diagram (np), deficiency number diagram (c), and (u) diagram. Given the point that example presented is related to the number of deficiencies of the product in a to aster manufacturing company (ismaelpur *et al.*, 2009), the control diagram (c) has been used; this diagram studies the number of deficiencies in a sample of (n) numbers. An impaired part may have several deficiencies. The main objective in providing this diagram is to control the number of existing deficiencies in an output sample product. In a way that sampling is conducted 25 to 30 times according to table (1). The volume of each sample must consist of at least 50 pieces, then, the number of deficiencies must be counted and the following computations should be conducted: center line, upper control limit and lower control limit.

$$lcl = \bar{c} - \sqrt{c}, ucl = \bar{c} + \sqrt{c}, cl = \bar{c} \quad (1)$$

Fuzzy data has been converted to definite figures using gravity center method.

$$\bar{c} = 65.65, \quad lcl = 65.65 - 3\sqrt{65.65}, \quad ucl = 65.65 + 3\sqrt{65.65}$$

**Table(1).samples and the number of faulty**

Number deficiency	sample	Number deficiency	sample	Number deficiency	sample	Number deficiency	sample	Number deficiency	sample
74	25	118	19	61	13	60	7	16/5	1
64	26	121	20	34	14	39	8	34	2
40/5	27	59	21	80/5	15	48	9	59	3
35	28	84	22	78/5	16	60	10	105/5	4
42/5	29	79/5	23	114	17	68	11	95/5	5
78	30	53	24	106	18	18	12	43/5	6

$$lcl = 41.34, ucl = 89.95$$

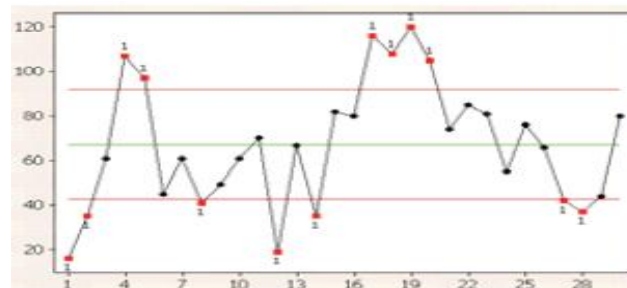


Figure 1. Control limit

Given the figure, the central line, upper control limit and lower control limit have been determined. Sample 1,2,4,5,8,12,14,17,18,19,20,27 and28 are out of control.

### To study the subject in fuzzy state:

**Fuzzy logic:** there are a lot of non-exact concepts around us which we express them in different fuzzy daily. Pay attention to this sentence, it is nice; no quantity is raised for weather goodness in order measure it, and this is a quality sense. Indeed, human brain defines and evaluates sentences given different factors and based on inductive thought, and determining their patterns in mathematics language and formulas is very difficult, though not impossible. Fuzzy logic is a new technology which replaces methods requiring advanced and complicated mathematics for designing and modeling a subsystem using linguistic quantities and the expert knowledge.

Professor lotfi zadeh argues that human does not need new data. Inputs, but he is able to conduct comparative control at a high level, thus, if feedback controls are designed in systems in a way that they can receive ambiguous data, these data could be used more simply and effectively in administration. Fuzzy logic believes that building the type of models should be sought that consider ambiguity as a part of the model system.

In Aristotle logic, there is a correct and an incorrect classification. All reports are correct or incorrect hence, the sentence, it is cold, is not basically a report in Aristotle model, since the degree of coldness is different for different individuals and this sentence is not basically always true or untrue. In fuzzy logic, there are sentences are to some extent true or untrue. For example, it is cold, is a fuzzy logic report whose accuracy is sometimes low and sometimes high. It is sometimes true and sometimes untrue and sometimes it is to some extent true. Fuzzy logic could be a fundamental basis for a new technology which has had enormous achievements so far.

### Fuzzy control:

In control diagrams especially descriptive ones that measure quality factors and parameters such as appearance, softness, color, etc., the feeling and tastes of the observer and inspector in expressing characteristics could be represented exactly in numerical quantity. In real state the rate (degree) of qualitative characteristic is stated in linguistic terms, but the diagrams of classic control di not have the capability to use these data. Fuzzy control diagram model for the purpose of using data that are stated by linguistic terms are applied (ismaelpur *et al.*, 2009).

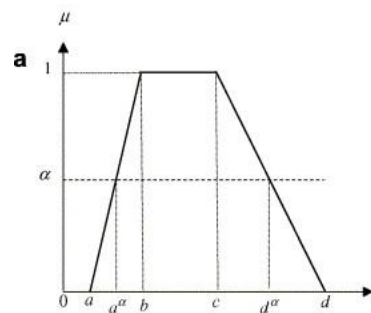
Fuzzy sets theory is the basis of fuzzy control diagrams. On the basis of this theory, two basic approaches have been developed in providing fuzzy control diagrams, fuzzy probability approach and membership approach (ismaelpur *et al.*, 2009).

### Membership approach:

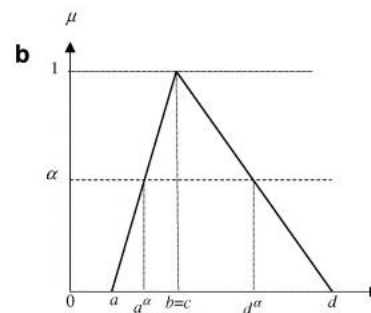
In this approach fuzzy subsets related to each linguistic quantity are added to each other in the sample and are divided into the measure. The result is a fuzzy set that may not be related to specific linguistic measure in the related term set, but it may be related to the quality level of mean in the sample. Therefore, a unit numerical quantity relate to the sample is achieved through converting total fuzzy set into resulting indicator quantity. This conversion is conducted according to one of the four methods mentioned above (Raz and Wong, 1990) and this method was called membership approach.

### Fuzzy probability approach:

In this approach, the indicator quantity is obtained directly from linguistic term or expressions related to observations the indicator sample quantity is computed as the mean of indicator quantities. This approach was called probability approach by (Raz and wang, 1990) and the control limit is obtained based on probability reasoning. After converting linguistic observations related to their indicator degrees or quantities, this approach resembles the application of variable control diagrams. Fuzzy state approach has been used as control in this article. For de fuzzy of the data and converting those into usual control diagram there are four methods: fuzzy state alpha fuzzy mode, fuzzy and mean and fuzzy mode and for selecting one of the methods there is no logical criterion or priority and it depended on user preference. considering the point that the subject under study is related to controlling the deficiency of a product , thus, in fuzzy state , when the number of deficiencies considers the mentality of the individuals or their lack of confidence, the triangle fuzzy figures as (a,b=c,d) and trapezoid fuzzy figures as (a,b,c,d). The central line of fuzzy control diagram (c) is observed in equation (2) and fig (2,3).



Figure(3).trapezium diagram



figure(2).fuzzy numbers with triangular diagram

$$\tilde{c}l = \left( \frac{\sum_{j=1}^m a_j}{m}, \frac{\sum_{j=1}^m b_j}{m}, \frac{\sum_{j=1}^m c_j}{m}, \frac{\sum_{j=1}^m d_j}{m} \right) = (\bar{a}, \bar{b}, \bar{c}, \bar{d}) \quad (2)$$

$$l\tilde{c}l = \tilde{c}l - 3\sqrt{\tilde{c}l} = (\bar{a}, \bar{b}, \bar{c}, \bar{d}) - 3\sqrt{(\bar{a}, \bar{b}, \bar{c}, \bar{d})} = (\bar{a} - 3\sqrt{\bar{a}}, \bar{b} - 3\sqrt{\bar{b}}, \bar{c} - 3\sqrt{\bar{c}}, \bar{d} - 3\sqrt{\bar{d}}) = (lcl_1, lcl_2, lcl_3, lcl_4) \quad (3)$$

$$u\tilde{c}l = \tilde{c}l + 3\sqrt{\tilde{c}l} = (\bar{a}, \bar{b}, \bar{c}, \bar{d}) + 3\sqrt{(\bar{a}, \bar{b}, \bar{c}, \bar{d})} = (\bar{a} + 3\sqrt{\bar{a}}, \bar{b} + 3\sqrt{\bar{b}}, \bar{c} + 3\sqrt{\bar{c}}, \bar{d} + 3\sqrt{\bar{d}}) = (ucl_1, ucl_2, ucl_3, ucl_4) \quad (4)$$

$\alpha$  -cut is a de fuzzy set that consists all elements whose membership degree is greater or equal to alpha.

$$a^\alpha = a + \alpha(b - a) \quad (5)$$

$$d^\alpha = d + \alpha(d - c) \quad (6)$$

In this way through developing alpha concept to control limit. Following formulas are obtained.

$$\tilde{c}l^\alpha = (\bar{a}^\alpha, \bar{b}, \bar{c}, \bar{d}^\alpha) = (cl_1^\alpha, cl_2, cl_3, cl_4^\alpha) \quad (7)$$

$$l\tilde{c}l^\alpha = \tilde{c}l^\alpha - 3\sqrt{\tilde{c}l^\alpha} = (\bar{a}^\alpha, \bar{b}, \bar{c}, \bar{d}^\alpha) - 3\sqrt{(\bar{a}^\alpha, \bar{b}, \bar{c}, \bar{d}^\alpha)} = (\bar{a}^\alpha - 3\sqrt{\bar{a}^\alpha}, \bar{b} - 3\sqrt{\bar{b}}, \bar{c} - 3\sqrt{\bar{c}}, \bar{d}^\alpha - 3\sqrt{\bar{d}^\alpha}) = (lcl_1^\alpha, lcl_2, lcl_3, lcl_4^\alpha) \quad (8)$$

$$u\tilde{c}l^\alpha = \tilde{c}l^\alpha + 3\sqrt{\tilde{c}l^\alpha} = (\bar{a}^\alpha, \bar{b}, \bar{c}, \bar{d}^\alpha) + 3\sqrt{(\bar{a}^\alpha, \bar{b}, \bar{c}, \bar{d}^\alpha)} = (\bar{a}^\alpha + 3\sqrt{\bar{a}^\alpha}, \bar{b} - 3\sqrt{\bar{b}}, \bar{c} - 3\sqrt{\bar{c}}, \bar{d}^\alpha + 3\sqrt{\bar{d}^\alpha}) = (ucl_1^\alpha, ucl_2, ucl_3, ucl_4^\alpha) \quad (9)$$

As stronger inspection and the mentality could be defined by quality management inspection has become more narrow or limited above the alpha amount when  $\alpha=1$ , the control diagram inclines toward classic state.

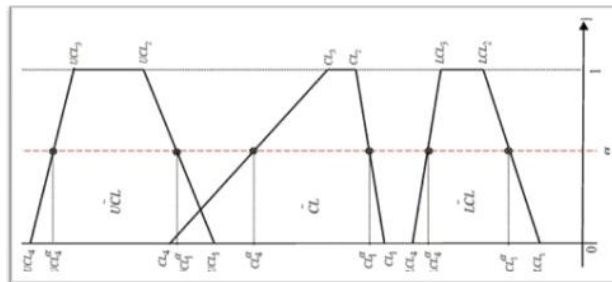


Figure 4. Control limit and  $\alpha = 1$  section

Some alternative methods for closed fuzzy control diagram based on developing fuzzy transition have been presented compared to other methods.

### FUZZY CONTROL DIAGRAM BASED ON FUZZY MIDDLE POINT AT $\alpha$ SURFACE:

Fuzzy middle point of  $\alpha$  at the surface of  $f_{mr}^\alpha$  is computed as follows: a and d are to ends of alpha section.

$$f_{mr}^\alpha = \frac{1}{2}(a^\alpha + d^\alpha) \quad (10)$$

This definition is used as following for de fuzzy of figures.

$$s_{mr,j}^\alpha = \frac{a_j^\alpha + d_j^\alpha}{2} = \frac{(a_j + d_j) + \alpha[(b_j - a_j) - (d_j - c_j)]}{2} \quad (11)$$

In this approach, control limit is defined as following.

$$s_{mr,j}^\alpha = \frac{a_j^\alpha + d_j^\alpha}{2} = \frac{(a_j + d_j) + \alpha[(b_j - a_j) - (d_j - c_j)]}{2} \quad (12)$$

$$lcl_{mr}^\alpha = cl_{mr}^\alpha - 3\sqrt{cl_{mr}^\alpha} \quad (13)$$

$$ucl_{mr}^\alpha = cl_{mr}^\alpha + 3\sqrt{cl_{mr}^\alpha} \quad (14)$$

Process control condition has been defined for each sample as follows and fig(5).

$$Processcontrol = \begin{cases} in-control, for \rightarrow lcl_{mr,j}^\alpha \leq s_{mr,j}^\alpha \leq ucl_{mr,j}^\alpha \\ out-of-control, otherwise \end{cases} \quad (15)$$

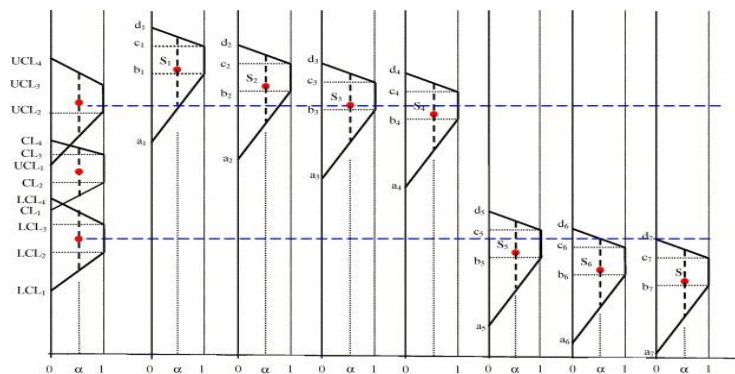


Figure 5. Samples s1,s2,s5,s6,s7 are out of control limit and s3,s4 are at control limit (Galbay and Kahraman, 2007).



**Fuzzy control diagram based on fuzzy mode conversation at alpha surface:**

The mode of alpha surface is a point that divides the membership function space of an alpha section into two parts fig (6).

$$s_{med.j}^{\alpha} = \frac{1}{4}(a_j^{\alpha} + b_j + c_j + d_j^{\alpha}) \tag{16}$$

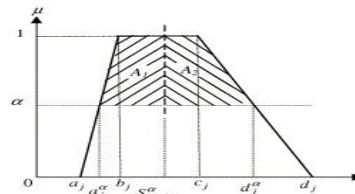


Figure 6. Picture of fuzzy mode(A1=A2). (Galbay and Kahraman, 2007)

Control limit s are defined as following and fig (7).

$$ucl_{med}^{\alpha} = cl_{med}^{\alpha} + 3\sqrt{cl_{med}^{\alpha}} \tag{17}$$

$$lcl_{med}^{\alpha} = cl_{med}^{\alpha} - 3\sqrt{cl_{med}^{\alpha}} \tag{18}$$

$$cl_{med}^{\alpha} = f_{med}^{\alpha}(c\tilde{l}) = \frac{1}{4}(cl_1^{\alpha} + cl_2 + cl_3 + cl_4^{\alpha}) \tag{19}$$

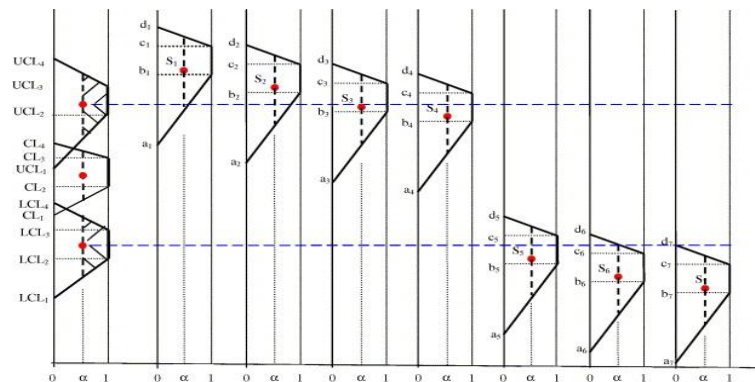


Figure 7 .j=(s1,s2,s3,s4,s5,s6,s7), is the result of two types of different decisions at surface  $\alpha = .5$  ,(s1,s2,s5,s6,s7) out of control ,and fuzzy mode at alpha surface out control limit , s3,s4 in control limit (Galbay and khahraman,2007)

Process control condition for each sample has been defined as follows:

$$Processcontrol = \begin{cases} in-control\ for \rightarrow lcl_{med}^{\alpha} \leq s_{med.j}^{\alpha} \leq ucl_{med}^{\alpha} \\ out-of-control, otherwise \end{cases} \tag{20}$$

**A NEW METHOD FOR FUZZY CONTROL DIAGRAM: DIRECT FUZZY APPROACH**

In this method for linguistic data is not convertible to representative values using fuzzy transition for the purpose of losing each type of information available in fuzzy samples. Thus, both of the samples and control limitations in fuzzy figures are toward the whole process shown. Limitations at a surface of fuzzy control  $u\tilde{c}l^{\alpha}, c\tilde{l}^{\alpha}, l\tilde{c}l^{\alpha}$  could be determined using fuzzy computations. as it has been shown in equations (2) to (9), decision-making about the point that whether this trend in control could be a percent of the sample which is confined inside  $u\tilde{c}l$  or  $l\tilde{c}l$ .

When a fuzzy sample is completely involved by fuzzy control limitation, this process is called in control. If a fuzzy sample omits the limitations of fuzzy control completely, this process is called out-of-control. Otherwise, it is a sample that some part of it has been included (inserted) by fuzzy control limitations. In this case, if the percentage of the area that remains inside the fuzzy control limit ( $\beta$ ), ( $j$  be equal or greater than the acceptable percent previously defined, ( $\beta$ )), then this trend could be called accepted or out-of-control. Otherwise, it could be declared as “not out-of-control. the probable decisions derived from DFA have been shown in fig (7). Parameters have been formed for determining the sample area out-of-control limit for each alpha section surface  $LCL_1, LCL_2, UCL_3, UCL_4, A, B, C, D$  and  $\alpha$  from control limit and fuzzy samples by lies,  $ab, cd, \mu_{l_3}, l_{l_4}, l_{l_1}, l_{l_2}$  computation of fuzzy sample area in fig(8). The sample area higher than upper control limit,  $A_{out}^u$  sample area and while falling below lower control unit,  $A_{out}^L$  the equations for computing  $A_{out}^u$  and  $A_{out}^L$  have been presented in appendix, the total area out of fuzzy control limit, is equal to the total areas under lower fuzzy control limit and higher than upper fuzzy control limit. The percent sample area has been computed in control limit as has been given in the following equation.

$$\beta_j^\alpha = \frac{s_j^\alpha - A_{out,j}^\alpha}{s_j^\alpha} \tag{21}$$

Place where  $s_j^\alpha$  is the sample area in a surface section .

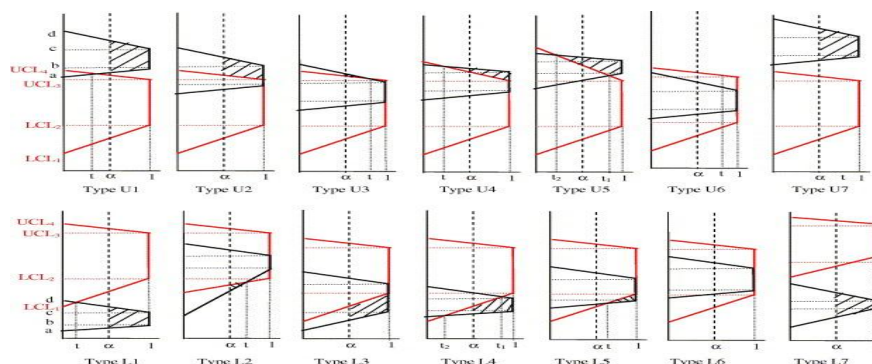


Figure 8. Picture of areas out of control limit at section surface (Galbay and kahraman,2007)

The flow chart for computing the area of a fuzzy sample (a,b,c,d) which is out of fuzzy control limit in fig(9) has been presented in appendix section, appendix(2-A). in comparison with methods, the use of fuzzy transition, is a much more accurate and reflective approach than both linguistic data and the control limits are converted into representative or indicator values for avoiding the loss of existing information in the samples.

**Numerical example:**

Samples have been provided according table (2) that have been presented in appendix section (3-A). Central limits, lower limit and upper limit along with fuzzy computations have been computed according to table (3).

**Table 3. Control limits and fuzzy calculation**

	Fuzzy number				Fuzzy transformation method		
	a	b	c	d	Fuzzy mode	middle ( $\alpha = 0.60$ )	median ( $\alpha = 0.60$ )
CL	52.8	59.1	72.167	75.167	[59.1,72.16]	65.18	65.41
LCL	26.618	33.615	49.104	54.368	[33.61,49.10]	40.95	41.14
UCL	74.599	82.163	97.65	102.35	[82.16,97.65]	89.4	89.67

Making decision on process control from each sample based on fuzzy state, fuzzy  $\alpha$  in the middle surface, average fuzzy  $\alpha$  at  $\alpha$  surface is based on table (4) which have been presented in appendix (4-A).

**THE APPLICATION OF DIRECT FUZZY APPROACH (DFA).**

The limitation of  $\alpha$  section fuzzy control has been presented in brief in table (5). The numerical results and decision – making process control based on DFA have been presented in table (6) appendix (5-A).

**Table 5. Control limitations for DFA(  $\alpha = 0.60$ )**

	$a^\alpha$	b	c	$d^\alpha$
CL	56.6	82.16	97.65	73.76
LCL	30.83	33.61	49.10	51.19
UCL	79.17	82.16	97.65	99.52

The general results obtained in this method which have been presented briefly in table (7) appendix (6-A). Show that it is clearly seen that some different decisions have been made. For example, in sample with fuzzy mode decision – making out of control, decision –making with fuzzy middle limit, out-of-control and decision-making with direct fuzzy approach is relatively out-of-control. DFA provides the possibility of linguistic decision-making such as relatively in-control most process control decision making levels may occur at different time intervals. For example for beta as it has been defined in equation (22).

$$\text{Processcontrol} = \begin{cases} \text{in-control, } .85 \leq \beta_j \leq 1 \\ \text{rather-in-control, } .60 \leq \beta_j \leq .85 \\ \text{rather-out-of-control, } .10 \leq \beta_j \leq .60 \\ \text{out-of-control, } 0 \leq \beta_j \leq .10 \end{cases} \tag{22}$$

Greater intervals for making decision on control process could be defined by decision-making mental power.

**CONCLUSION**

As it is observed in table no (6), decision-making for process control with different approaches contains slight differences that considering the loss of information from fuzzy samples. They have been turned into brittle figures. If the indicator of fuzzy figure from linguistic variables is symmetric, fuzzy state, middle fuzzy methods of  $\alpha$  converted sample will be equal to each other and thus controls decisions are the same fuzzy average is another method that could be useful for converting linguistic data into a representative quantity. In general, fuzzy state and middle fuzzy methods are easier than fuzzy average and fuzzy middle in terms of computation specially, when the membership function is nonlinear. Although mode fuzzy state may lead to a biased result one-sided. When the membership function is asymmetric if the surface under membership function is considered, fuzzy average is more appropriate ambiguous criterion. It can be said that de fuzzy with different methods in some cases has led to the problem that samples face different result in being accepted or rejected that is indicator of the loss of fuzzy information. Through direct fuzzy approach, the degree of strictness and accuracy in control limit could be adjusted with higher accuracy through adjusting the quantities. Also, other ranking approaches could be used for comparison. in comparing classic control and fuzzy control is along with supporting linguistic variables, high reflectivity, data based on expert observations, uncertain data, multi-characteristic control and classic control is along with classifying based on zero and one, limited reflectivity, reliance on past data, certain data and controlling just one characteristic. Thus, the accuracy of fuzzy control method is higher than classic method.



## APPENDEXXS

### Appendix (1-A):

The equations for calculating the out of control

equation( A.U1 )

$$A_{out}^U = \frac{1}{2} [(d^z - UCL_4^z) + (d^t - UCL_4^t)](\max(t - \alpha, 0)) + \frac{1}{2} [(d^z - a^z) + (c - b)](\min(1 - t, 1 - \alpha))$$

wherever

$$t = \frac{UCL_4 - a}{(b - a) + (c - b)} \quad \text{and} \quad z = \max(t, \alpha)$$

equation ( A.U2 )

$$A_{out}^U = \frac{1}{2} [(d^z - UCL_4^z) + (c - UCL_3)](1 - \alpha)$$

equation ( A.U3 )

$$A_{out}^U = \frac{1}{2} (d^z - UCL_4^z)(\max(t - \alpha, 0))$$

wherever

$$t = \frac{UCL_4 - d}{(UCL_4 - UCL_3) - (d - c)}$$

equation ( A.U4 )

$$A_{out}^U = \frac{1}{2} [(c - UCL_3) + (d^z - UCL_4^z)](\min(1 - t, 1 - \alpha))$$

wherever

$$t = \frac{UCL_4 - d}{(UCL_4 - UCL_3) - (d - c)} \quad \text{and} \quad z = \max(t, \alpha)$$

equation ( A.U5 )

$$A_{out}^U = \frac{1}{2} [(d^{z_2} - UCL_4^{z_2}) + (d^{t_1} - UCL_4^{t_1})](\min(\max(t_1 - \alpha, 0), t_1 - t_2)) + \frac{1}{2} [(d^{z_1} - a^{z_1}) + (c - b)] \times (\min(1 - t_1, 1 - \alpha))$$

wherever

$$t_1 = \frac{UCL_4 - a}{(b - a) + (UCL_4 - UCL_3)}, \quad t_2 = \frac{UCL_4 - d}{(UCL_4 - UCL_3) - (d - c)}$$

$$z_1 = \max(\alpha, t_1), \quad \text{and} \quad z_2 = \max(\alpha, t_2)$$

equation ( A.U6 )

$$A_{out}^U = 0$$

equation ( A.U7 )

$$A_{out}^U = \frac{1}{2} [(d^z - a^z) + (c - b)](1 - \alpha)$$

equation ( A.L1 )

$$A_{out}^L = \frac{1}{2} [(LCL_1^z - a^z) + (LCL_1^t - a^t)](\max(t - \alpha, 0)) + \frac{1}{2} [(d^z - a^z) + (c - b)](\min(1 - t, 1 - \alpha))$$

wherever

$$t = \frac{d - LCL_1}{(LCL_2 - LCL_1) + (d - c)} \quad \text{and} \quad z = \max(\alpha, t)$$

equation ( A.L2 )

$$A_{out}^L = \frac{1}{2} [(d^z - a^z) + (c - b)](1 - \alpha)$$

equation ( A.L3 )

$$A_{out}^L = \frac{1}{2} [(LCL_1^z - a^z) + (LCL_2 - b)](1 - \alpha)$$

equation ( A.L4 )

$$A_{out}^L = \frac{1}{2}[(LCL_1^{z_2} - a^{z_2}) + (LCL_1^{t_1} - a^{t_1})](\min(\max(t_1 - \alpha, 0), t_1 - t_2))$$

$$+ \frac{1}{2}[(d^{z_1} - a^{z_1}) + (c - b)](\min(1 - t, 1 - \alpha))$$

wherever

$$t_1 = \frac{d - LCL_1}{(LCL_2 - LCL_1) + (d - c)}, \quad t_2 = \frac{a - LCL_1}{(LCL_2 - LCL_1) - (b - a)}$$

$$z_1 = \max(\alpha, t_1), \quad \text{and} \quad z_2 = \max(\alpha, t_2)$$

equation ( A.L5 )

$$A_{out}^L = \frac{1}{2}[(LCL_1^z - a^z) + (LCL_2 - b)](\min(1 - t, 1 - \alpha))$$

wherever

$$t = \frac{a - LCL_1}{(LCL_2 - LCL_1) - (b - a)}, \quad \text{and} \quad z = \max(\alpha, t)$$

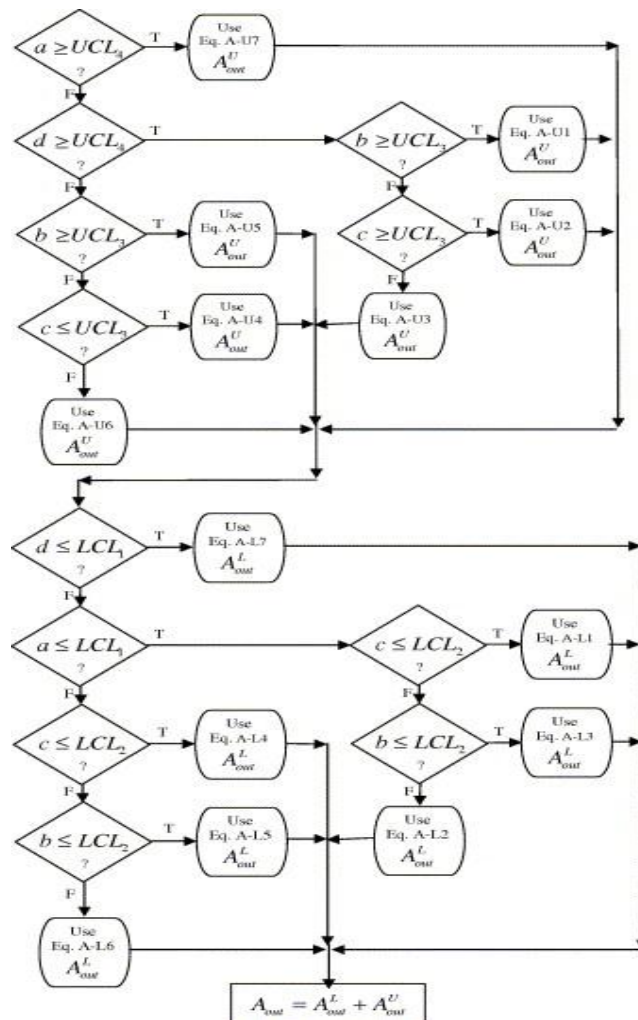
equation ( A.L6 )

$$A_{out}^L = 0$$

equation ( A.L7 )

$$A_{out}^L = \frac{1}{2}[(d^z - a^z) + (c - b)](1 - \alpha)$$

## Appendix (2-A)



**Figure( 9). Flowchart for determining the area of the sample fuzzy (a, b, c, d) are out of control limits are fuzzy. (Galbay and Kahraman, 2007)**

**Appendix (3-A)**

**Table( 2): Limit values of fuzzy control, fuzzy median and average phase (Ismaelpour *et al.*, 2009)**

a	b	c	d	sample
11	12	21	25	1
22	28	40	44	2
45	52	66	70	3
93	99	112	116	4
83	89	102	106	5
35	37	50	54	6
48	54	66	70	7
26	32	46	50	8
35	42	54	58	9
48	54	66	70	10
55	61	75	79	11
6	12	24	28	12
49	55	67	71	13
22	28	40	44	14
68	74	87	91	15
66	72	85	89	16
99	107	121	125	17
92	99	113	117	18
103	111	125	129	19
106	114	128	132	20
45	52	66	70	21
72	78	90	94	22
67	73	86	90	23
38	46	60	64	24
59	67	81	85	25
49	57	71	75	26
28	34	47	51	27
21	28	42	46	28
30	36	49	53	29
63	71	85	89	30

**Appendix (4-A):**

**Table(4).decision in fuzzy state, fuzzy middle and fuzzy average (  $\alpha = 0.60$  )**

sample	$f_{mod.j}^{\alpha=0.60}$	$\beta J$	decision	$f_{mod.j}^{\alpha=0.60}$	$f_{mr.j}^{\alpha=0.60}$	$f_{mr.j}^{\alpha=0.60}$	decision	$f_{med.j}^{\alpha=0.60}$	$f_{med.j}^{\alpha=0.60}$	decision
1	12	21	0	Out of control	17.1	Out of control	16.8	Out of control		
2			.4679	Rather out of control	33.6	Out of control	33.8			
3	52	66	1	Under control	58.4	Under control	58.7	Under control		
4	99	112	0	Out of control	105.1	Out of control	105.3	Out of control		
5			.6655	Rather out of control	95.1	Out of control	95.3			
6	37	50	1	Under control	43.9	Under control	43.7	Under control		
7	54	66	1	Under control	59.6	Under control	59.8	Under control		
8			.1153	Rather out of control	38.6	Out of control	38.8			
9	42	54	1	Under control	47.4	Under control	47.7	Under control		
10	54	66	1	Under control	59.6	Under control	59.8	Under control		
11	61	75	1	Under control	67.6	Under control	67.8	Under control		
12	12	24	0	Out of control	17.6	Out of control	17.8	Out of control		
13	55	67	1	Under control	60.6	Under control	60.8	Under control		
14			.4679	Rather out of control	33.6	Out of control	33.8			
15	74	87	1	Under control	80.1	Under control	80.3	Under control		
16	72	85	1	Under control	78.1	Under control	78.3	Under control		
17	107	121	0	Out of control	113.2	Out of control	113.6	Out of control		
18	99	113	0	Out of control	105.4	Out of control	105.7	Out of control		
19	111	125	0	Out of control	117.2	Out of control	117.6	Out of control		
20	114	128	0	Out of control	120.2	Out of control	120.6	Out of control		
21	52	66	1	Under control	58.4	Under control	58.7	Under control		
22	78	90	1	Under control	83.6	Under control	83.8	Under control		
23	73	86	1	Under control	79.1	Under control	79.3	Under control		
24	46	60	1	Under control	52.2	Under control	52.6	Under control		
25	67	81	1	Under control	73.2	Under control	73.6	Under control		
26	57	71	1	Under control	63.2	Under control	63.6	Under control		
27	34	47	1	Under control	40.1	Out of control	40.3	Under control		
28			.4010	Rather under control	34.4	Under control	34.7			
29	36	49	1	Under control	42.1	Under control	42.3	Under control		
30	71	85	1	Under control	77.2	Under control	77.6	Under control		

**Appendix (5-A):**

**Table (6). The decision on the direct fuzzy approach ( $\alpha = 0.60$ ), ( $\beta = 0.70$ )**

<i>sample</i>	$d^a$	$b$	$c$	$d^a$	Of area	Sample area	$\beta J$	<i>decision DFA</i>
1	11.6	12	21	22.6	4	4.8	16.66667	Rather out of control
2	25.6	28	40	41.6	2.176	5.6	61.14286	Rather under control
3	49.2	52	66	67.6	0	6.48	100	Under control
4	96.6	99	112	113.6	0	6	100	Under control
5	86.6	89	102	103.6	0	6	100	Under control
6	36.2	37	50	51.6	0	5.68	100	Under control
7	51.6	54	66	67.6	0	5.68	100	Under control
8	29.6	32	46	47.6	0.576	6.4	91	Under control
9	39.2	42	54	55.6	0	5.68	100	Under control
10	51.6	54	66	67.6	0	6	100	Under control
11	58.6	61	75	76.6	0	6.4	100	Under control
12	9.6	12	24	25.6	0	5.6	100	Under control
13	52.6	55	67	68.6	0	5.6	100	Under control
14	25.6	28	40	41.6	5.6	5.6	0	Out of control
15	71.6	74	87	88.6	0	6	100	Under control
16	69.6	72	85	86.6	0	6	100	Under control
17	103.8	107	121	122.6	0	6.56	100	Under control
18	96.2	99	113	114.6	0	6.68	100	Under control
19	107.8	111	125	126.6	0	6.56	100	Under control
20	110.8	114	128	129.6	0	6.56	100	Under control
21	49.2	52	66	67.6	0	6.48	100	Under control
22	75.6	78	90	91.6	0	5.68	100	Under control
23	70.6	73	86	87.6	0	6	100	Under control
24	42.8	46	60	61.6	0	6.56	100	Under control
25	63.8	67	81	82.6	0	6.56	100	Under control
26	53.8	57	71	72.6	0	6.56	100	Under control
27	31.6	34	47	48.6	0	6	100	Under control
28	25.2	28	42	43.6	2.248	6.48	65.30864	Rather under control
29	33.6	36	49	50.6	0	6	100	Under control
30	67.8	71	85	86.6	0	6.4	100	Under control



**Appendix (6-A):**

**Table (7): Comparison of alternative methods: fuzzy state, fuzzy intermediate, average fuzzy and DFA ( $\alpha = 0.60$ ), (Esmaelpour *et al.*, 1388).**

$J$	<i>decision classic</i>	<i>decision <math>f_{mod}^{\alpha=0.60}</math></i>	<i>decision <math>f_{mr.j}^{\alpha=0.60}</math></i>	<i>decision <math>f_{med.j}^{\alpha=0.60}</math></i>	DFA ( $\alpha = 0.60$ ) <i>decision</i>
1	Out of control	Out of control	Out of control	Out of control	Rather out of control
2	Under control	Rather out of control	Out of control	Out of control	Rather under control
3	Out of control	Under control	Under control	Under control	Under control
4	Out of control	Out of control	Out of control	Out of control	Under control
5	Under control	Rather out of control	Out of control	Out of control	Under control
6	Under control	Under control	Under control	Under control	Under control
7	Out of control	Under control	Under control	Under control	Under control
8	Under control	Rather out of control	Out of control	Out of control	Under control
9	Under control	Under control	Under control	Under control	Under control
10	Under control	Under control	Under control	Under control	Under control
11	Out of control	Under control	Under control	Under control	Under control
12	Under control	Out of control	Out of control	Out of control	Under control
13	Out of control	Under control	Under control	Under control	Under control
14	Under control	Rather out of control	Out of control	Out of control	Out of control
15	Under control	Under control	Under control	Under control	Under control
16	Out of control	Under control	Under control	Under control	Under control
17	Out of control	Out of control	Out of control	Out of control	Under control
18	Out of control	Out of control	Out of control	Out of control	Under control
19	Out of control	Out of control	Out of control	Out of control	Under control
20	Under control	Out of control	Out of control	Out of control	Under control
21	Under	Under control	Under	Under control	Under control

$J$	decision classic	decision $f_{\text{mod}}^{\alpha=0.60}$	decision $f_{\text{mr.j}}^{\alpha=0.60}$	decision $f_{\text{med.j}}^{\alpha=0.60}$	DFA ( $\alpha = 0.60$ ) decision
	control		control		
22	Under control	Under control	Under control	Under control	Under control
23	Under control	Under control	Under control	Under control	Under control
24	Under control	Under control	Under control	Under control	Under control
25	Under control	Under control	Under control	Under control	Under control
26	Out of control	Under control	Under control	Under control	Under control
27	Out of control	Under control	Out of control	Under control	Under control
28	Under control	Rather under control	Under control	Out of control	Rather under control
29	Under control	Under control	Under control	Under control	Under control
30	Out of control	Under control	Under control	Under control	Under control

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